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Interactive horizontal load model for pedestrians crossing footbridges

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In recent years several theories have been put forward in relation to lateral forces imparted on bridges from crowd movements. It is now widely accepted that the interaction between the crowd and the structure, particularly when the crowd pacing frequency is close to the lateral natural frequency, is the major factor determining the lateral response. However, very little work has been done with individual pedestrians in order to determine the relationship between the lateral force induced by a single pedestrian and the structural response. Equally, most literature concerning lateral forces induced from walking is based on results from fixed force plate tests and hence no assessment of any interaction is made.

This paper examines the lateral response of a GRP (glass-fibre reinforced polymer) cable-stayed footbridge to individual pedestrian crossings at a range of pacing frequencies. Two lateral load modelling approaches are considered. The first approach, derived from back analysis of the measured bridge response, was found to be ineffective in predicting the measured response accurately. A second modelling approach, incorporating a spring-damper to represent a moving pedestrian, which thereby accounts for the interaction between the structural response and the mass, stiffness and damping characteristics of the body of the traversing pedestrian, is demonstrated to be more effective in simulating the bridge response.

Keywords: Bridges, Human–Structure Interaction, Dynamic response

1. Introduction

Modern pedestrian bridges are becoming increasingly daring in design. Footbridges are now being designed not just for functionality but also to serve as major landmarks in their urban environments. Alongside advances in understanding of the structural behaviour on behalf of the designer, the use of new materials have led to greater spans and more slender structures being achieved than ever before. Examples of high profile new bridges all built around the year 2000 and all occupying focal points in their respective cities are the Millennium Bridges in Dublin and London, and the Pont du Solferino in Paris.

As footbridges have become more slender and lighter than ever before, new challenges have been created that are not properly addressed in the major codes of practice currently in use. The most recently prominent among these are lateral vibrations induced by pedestrians walking across the bridge. In one of the earliest references to lateral loads, Tilden (1913) argued that the magnitude of the lateral force exerted by a single pedestrian is so small that this type of loading is not critical. In their study on pedestrian-induced

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loading Bachmann and Ammann (1987) confirmed that laterally induced forces are small, less than 10%, of the vertical loads. Tilden (1913) argued further that significant horizontal loading would only arise when there was more than one pedestrian on the bridge structure at any moment in time and only then if they synchronized their movements, which he considered improbable. This was widely accepted to be the case until the Millennium Bridge in London (Dallard *et al*. 2001) was opened on 10 June 2000. It was estimated that 80 000 people crossed the bridge on that day. The bridge was closed soon after due to reports of excessive swaying.

This was not the first occasion in which lateral vibration of a footbridge had been observed—Bachmann and Ammann (1987) report a case documented by Petersen (1972), in which a steel footbridge with a fundamental lateral frequency of approximately 1.1 Hz, vibrated strongly in the lateral direction during the opening ceremony, when 300–400 persons crossed the bridge. Fujino (1993) reported similar lateral vibrations in a cable stayed bridge with a steel box girder deck, with a first lateral frequency of the deck of 0.9 Hz. These vibrations were induced when large crowds crossed the structure after boat races, resulting in up to 2000 pedestrians occupying the 180 m span of the bridge at any one time.

Following an extensive period of testing and research (Fitzpatrick 2001) on the Millennium Bridge in London, a substantial finding was that the lateral force exerted by a pedestrian on the footbridge is correlated to and proportional to the response of the bridge, i.e. the interaction between the human and the structure is important. This research further indicated that any bridge with a lateral natural frequency of close to 1 Hz was susceptible to excitation from groups of pedestrians, irrespective of the structural form or construction material.

Current international design standards use non-interactive vertical load models only to account for pedestrians on footbridges. In these models the force applied does not depend on the bridge movement. Archbold (2004) and Fanning *et al*. (2005) demonstrated the enhanced benefit of interactive vertical load models in simulating the response of Aberfeldy Footbridge, a lightweight flexible footbridge, to traversing pedestrians. Furthermore Archbold (2004) developed a new interactive lateral load model which was tested against the lateral data measured on Aberfeldy Footbridge. Following earlier investigations on the same bridge, Pavic *et al*. (2000), through back analysis of the response of the footbridge to single pedestrian crossings, reported a lateral force that varied sinusoidally in time with a frequency equal to half of the pacing frequency. These two load modelling approaches are compared in terms of the correspondence of simulated bridges responses to measured bridge responses in this paper.

2. Aberfeldy Footbridge—testing and modelling

Aberfeldy Footbridge (figure 1) is a cable-stayed footbridge over the River Tay on a golf course in Aberfeldy, Scotland. Its main span is 63 m long and it has two side spans of approximately 25 m each. It is constructed almost entirely from a glass-fibre reinforced polymer (GRP) composite material.

The results of modal testing and pedestrian crossing tests on Aberfeldy Footbridge, undertaken in the summer of 2000, were subsequently reported by Pavic *et al*. (2000). Analysis of measured frequency response functions yielded 14 measured mode shapes and their corresponding natural frequencies and modal damping ratios. The first nine of these are summarized in table 1 (where 'L' and 'V' indicate lateral and vertical modes of vibration, respectively).

Pavic *et al*. (2000) also recorded vertical and horizontal acceleration measurements for approximately 100 controlled pedestrian crossings of the footbridge. The test subjects involved in the tests and the principal corresponding pacing frequencies simulated are listed in table 2. Where nominal pacing rates were used the pacing rates are designated 'S', 'N' and 'F' for slow, normal and fast walking, respectively, with the actual pacing rate achieved in parentheses. All other pacing rates were controlled with a metronome.

The measured lateral accelerations, at mid-span for test subject 1 (TS1), walking at a pacing frequency of 1.5 Hz, are plotted in figure 2. A 10 s root-mean-squared (rms) of this acceleration history is also included in the figure—the maximum 10 s rms acceleration is approximately 0.075 m/s^2. In this paper measured and simulated accelerations are compared on the basis of 10 s rms acceleration levels.

During testing each test subject traversed the bridge twice. The lateral response was found by Pavic *et al*. (2000)

Figure 1. Aberfeldy Footbridge.

Table 1. First nine measured mode shapes and frequencies. 'L' and 'V' indicate lateral and vertical modes of vibration, respectively.

Mode shape	Natural frequency (Hz)
L1	0.98
V1	1.52
V2	1.86
V3	2.49
L2	2.73
V4	3.01
V5	3.5
V6	3.91
V7	4.40

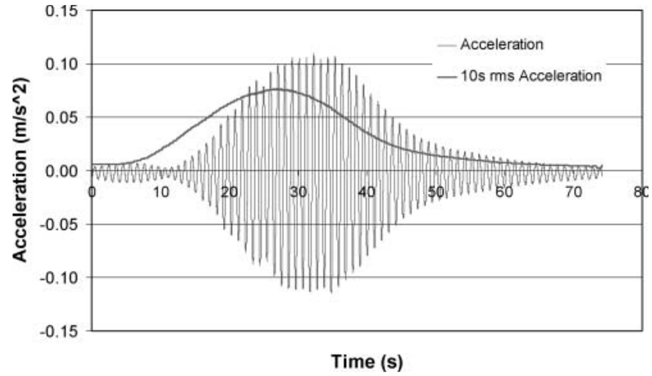

Figure 2. TS1—test 1—measured lateral accelerations versus time.

Table 2. Test subjects and pacing rates. The pacing rates are designated 'S', 'N' and 'F' for slow, normal and fast walking, respectively, with the actual pacing rate achieved in parentheses.

Test subject	Height (m)	Mass (kg)	Pacing frequencies (Hz)
TS1	1.90	104	1.4, 1.5, 1.6, 1.7, 1.8, 1.9, S(1.72), N(1.96), F(2.14)
TS2	1.79	88	1.90, S(1.57), N(1.80), F(1.94)
TS3	1.73	73	1.9, S(1.44), N(1.70), F(1.98)
TS4	1.78	88	1.9, S(1.54), N(1.91), F(2.08)
TS5	1.93	86	1.4, 1.5, 1.6, 1.7, 1.8, 1.9 S(1.57), N(1.9), F(2.10)
TS7	1.73	67	1.9, S(1.90), N(2.03), F(2.11)
TS8	1.83	79	1.4, 1.6, 1.8, 1.9, 2.0
TS9	1.73	86	1.4, 1.6, 1.8, 1.9, 2.0

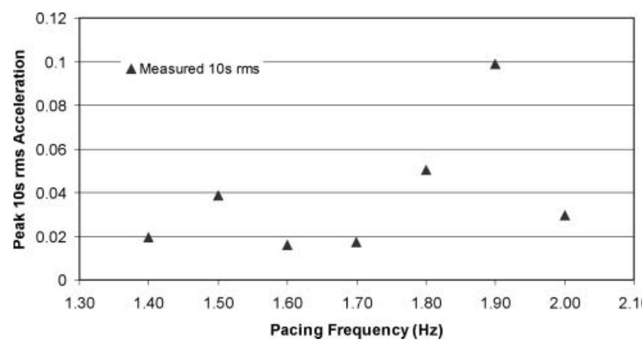

Figure 3. TS1—average peak 10 s rms lateral accelerations versus pacing frequencies.

to be a function of the frequency—this is consistent with the response trend in the vertical direction also. The average peak 10 s rms accelerations for TS1 are plotted against pacing frequency in figure 3. The maximum response was recorded when the pacing frequency was twice the first lateral frequency, i.e. at 1.9 Hz. There was also a less pronounced attenuation of lateral response at a pacing frequency of 1.5 Hz, which is the frequency of the first vertical mode of vibration of the bridge.

A further interesting feature of the lateral responses was the lack of correlation between bridge response and test subject mass. The peak lateral (L) 10 s rms accelerations, for TS1, TS5 and TS9 traversing the bridge at a pacing rate of 1.9 Hz, are summarized in table 3. Whilst the lateral responses for two test subjects, TS1 and TS5, are consistent with each other the maximum acceleration for TS1 is 0.099 m/s^2 compared with 0.0265 m/s^2 for TS5. In contrast the vertical (V) accelerations are in keeping with the mass ratios of the test subjects. Additionally, test subjects TS5 and TS9, both having the same mass, result in significantly different peak accelerations responses in the lateral direction.

Subsequent to this set of tests, Archbold (2004) undertook a series of simulations aimed at developing interac-tive and vertical load models for single pedestrian loading. As part of this process a three-dimensional finite element model, using linear beam elements to represent the bridge deck, was developed in ANSYS (V7.1). The finite element model was manually updated and validated through comparison with the measured modal properties. The measured and predicted natural frequencies are compared in table 4. This validated model was subsequently used to examine vertical and lateral load-models.

3. Horizontal forces from walking

The lateral force imparted from a person walking on a rigid surface can be measured using a force plate—a representative lateral force – time function for consecutive footsteps is shown as a solid line in figure 4. Equally, the frequency of lateral loading is half that of vertical loading as it coincides with the force exerted by the body weight on one foot (either right or left) and not the impulse exerted sequentially by both feet as in the vertical case.

Table 3. Peak 10 s rms lateral and vertical accelerations for TS1 and TS5 at 1.9 Hz pacing rate.

		Peak 10 s rms acceleration (m/s^2)		
Test subject	Mass (kg)	Test 1	Test 2	Average
TS1(L)	104	0.1061	0.0919	0.099
TS5(L)	86	0.0294	0.0235	0.0265
TS9(L)	86	0.1193	0.1217	0.1205
TS1(V)	104	0.162	0.150	0.156
TS5(V)	86	0.136	0.150	0.143

Table 4. Measured and predicted natural frequencies.

	Natural frequency (Hz)		
Mode shape	Measured	Predicted	% Error
L1	0.98	0.98	0.0
V1	1.52	1.54	+1.3
V2	1.86	1.85	−0.5
V3	2.49	2.49	0.0
L2	2.73	2.80	+2.6
V4	3.01	2.99	−0.7
V5	3.5	3.51	+0.3
V6	3.91	3.90	−0.2
V7	4.40	4.39	−0.2

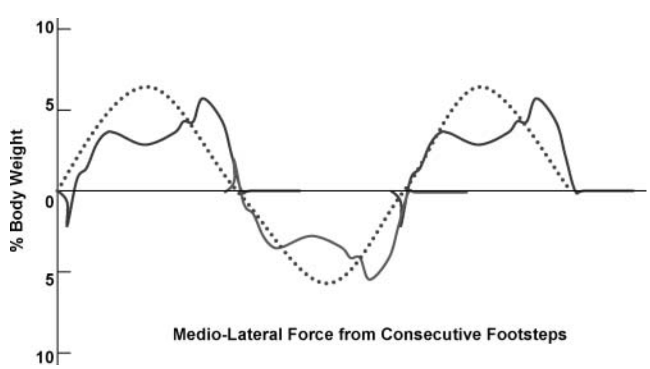

Figure 4. Medio-lateral force from consecutive footsteps.

Pavic *et al.* (2000), through back analysis of the response of Aberfeldy Footbridge to single pedestrian crossings, reported a lateral force that varied sinusoidally in time with a frequency half of the pacing frequency. This force, the dashed sinusoidal function in figure 4, was expressed mathematically as

$$F_{\mathrm{L}}(t) = L_{\mathrm{F}} G \sin(\pi f_{\mathrm{s}} t) \quad (1)$$

where $F_{\mathrm{L}}(t)$ = lateral force applied by person walking, G = weight of person, f_{s} = pacing frequency, t = time, L_{F} = load factor, established as 4% of body weight (0.04).

A feature of the above lateral load model is that the load factor is not pacing frequency dependent and also no account of the variability in pedestrians gait is accounted for—such a load model would not be expected to account for the variability in response measured for TS5 and TS9, two test subjects of the same mass producing bridge accelerations differing by a factor of four.

Simpson and Jiang (1999) reported tests that revealed that foot landing position influenced the lateral force applied by a pedestrian. They categorized their test participants into categories of 'toe-in', 'neutral' and 'toe-out' depending on their foot landing position during straight-line walking as shown in figure 5. 'Toe-out' participants were found to impart a lateral force up to 4 times that of 'toe-in' participants.

4. Lateral force simulation

Two approaches—a moving force model and a moving spring-damper model—for simulating lateral load effects due to the traversing test subjects were considered. In respect of the spring-damper model both positive and negative spring stiffnesses were considered.

4.1 *Sinusoidal lateral force*

The force produced by a test subject was simulated by a moving point force using the force – time function described by Pavic *et al.* (2000) in equation (1) above. A value of 0.04 was used for L_{F}, i.e. the magnitude of the lateral force varied sinusoidally with a maximum value of 4% of the body weight of the pedestrian. This force was thus applied to a point element in contact with the bridge and a displacement was applied to the point element to simulate the pedestrian crossing the bridge. The time to traverse the bridge was controlled in each transient analysis to ensure the appropriate pacing frequency was captured.

4.2 *Spring-damper model*

The argument for using a spring-damper model is based on humans' response to lateral movements. Humans, on detection of lateral movement, will tend to adjust their walking pattern to 'go with' the movement rather than try to resist it, thus stabilizing their body and minimizing shock and disruption to their trunk and head movements (Gard and Childress 2001, Menz *et al.* 2003). The spring-damper approach seeks to provide a mechanism for adapting the applied force based on the lateral movement of the structure.The moving spring-damper model considered is shown in figure 6. Contact is specified between the contact node and the bridge while the opposite end of the spring-damper model is held in position laterally at the grounding node. The lateral dynamic force – time function described in

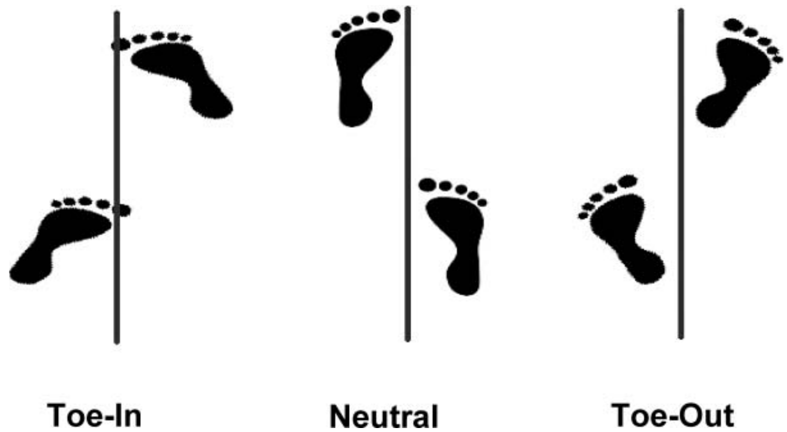

Figure 5. Foot landing position (FLP) categories (Simpson and Jiang 1999).

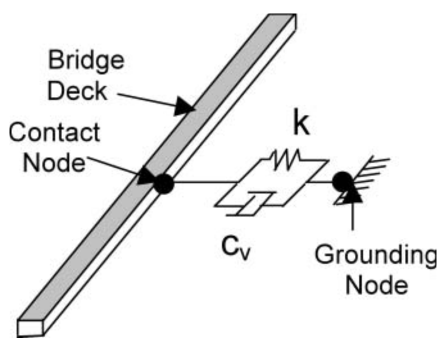

Figure 6. The spring-damper model.

equation (1) was applied to the contact node, with an initial value of 0.04 for L_F, and as in the case of the laterally applied point load the spring-damper model was displaced along the bridge at an appropriate velocity to achieve the required pacing rate.

The spring-damper model used required input of values for k, the spring stiffness and damping c_v. The damping portion of the element contributes only damping coefficients to the structural damping matrix with the damping force (F_x) computed as:

$$Fx = -cv\frac{\mathrm{d}ux}{\mathrm{d}t} \tag{2}$$

where u_x is the relative linear displacement between nodes and c_v is the damping coefficient given by

$$c_v = (c_v)_1 + (c_v)_2 v \tag{3}$$

and v is the relative velocity between the two nodes of the element calculated in the previous sub-step. The second damping coefficient $(c_v)_2$ is used to produce a velocity-dependent nonlinear damping effect.

There is a wide range of values reported in the biomechanical literature for human leg stiffness and the amount of damping available in the human body. Zhang *et al.* (2000) derived values for human leg stiffness of 28.5 kN/m and a viscous damping coefficient, c_v, of 950 Ns/m. Arampatzis *et al.* (1999), meanwhile, reported leg stiffness values between 25.29 and 35.21 kN/m for people running between speeds of 2.61 and 6.59 m/s. Ferris *et al.* (1999) studied the effects on leg stiffness of runners when changing to a running surface of different stiffness properties to the previous one on which they ran. They report values for leg stiffness of between 6.9 and 10.0 kN m^{-1}. Farley and Gonzalez (1996) studied the relationship between stride frequency and leg stiffness in runners and reported leg stiffness values of between 7.0 and 16.3 kN/m between the lowest and highest pacing frequencies, which were reported in terms of variation from comfortable pacing frequencies for a running speed of 2.5 m/s. Whilst the above refer to vertical leg stiffness they represent a reasonable indicative range for initial input values to the spring-damper model. A value of 15 kN/m was used initially for the spring stiffness, with 600 Ns/m and 6000 Ns^2/m^2 being used for $(c_v)_1$ and $(c_v)_2$, respectively.

4.3 *Negative stiffness spring-damper model*

An interesting finding of the *in situ* tests on the bridge structure was the pacing frequency at which the peak response occurred—1.9 Hz. The measured lateral natural frequency (L1 mode) of the bridge was 0.98 Hz—on this basis the expectation was that a pacing frequency of 1.96 Hz would produce the maximum lateral bridge response due to the fact that the lateral loading is applied at half the pacing frequency. For most test subjects, 2.0 Hz was the closest designated pacing frequency to 1.96 Hz and it was expected that this pacing rate would produce the greatest structural response. However, the tests indicate that in all cases the peak response occurred for a pacing rate of 1.9 Hz with the responses for a pacing frequency of 2.0 Hz being somewhat lower.

Numerical simulations demonstrated that this could not be explained merely by the additional mass of the traversing pedestrian on the bridge. Equally, initial simulations using the spring-damper model described above also showed a slight shift in the expected pacing frequency at which the maximum response occurred—but in the opposite direction to that discernible from test results. Coupled with the premise that a person walking on a moving lateral surface will tend to move with that surface, a spring-damper model with a negative spring stiffness was also considered.

The equation representing the lateral load applied in this load model is given below:

$$F_L(t) = (-0.05 f_s + 0.12) P_f G \sin(\pi f_s t) \tag{4}$$

It is similar to equation (1) used for both prior load models considered except that it includes a frequency-dependent lateral load factor L_F given by:

$$L_F = (-0.05 f_s + 0.12) P_f \tag{5}$$

where f_s = pacing frequency, P_f = foot landing position factor following Simpson and Jiang (1999).

5. Comparison of simulation and test results

The peak 10 s rms accelerations from the finite element simulations and the measured bridge accelerations for TS1 and TS5 are shown in figures 7 and 8. The point force simulations are found to substantially overestimate the bridge accelerations and equally the frequencies at which peak responses occur are not consistent between the simulations and the measurements. The peak simulated response was at 1.96 Hz, which was to be expected as this was exactly twice the fundamental lateral natural frequency of 0.98 Hz. However, the peak measured response occurred at 1.90 Hz, whilst a value closer to 2.0 Hz would have been expected.

The results achieved using the spring-damper model for TS1 are shown, in addition to the averaged measured results and the results of the lateral sinusoidal force, in figure 9. The effect of the spring in the load model is to cause the peak predicted response to occur at 2.0 Hz—a shift from the previous prediction of 1.96 Hz. This shift in peak response, however, is in the opposite direction to the measured peak shift. Relative to the lateral sinusoidal force model the level of the 10 s rms acceleration is improved.

Using the negative-stiffness-spring-damper model the shift in the peak response was found to be consistent with

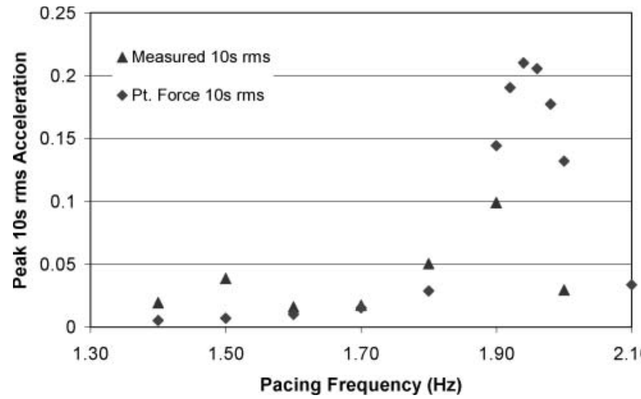

Figure 7. Peak 10 s rms accelerations for TS1—lateral sinusoidal force. Pt, point.

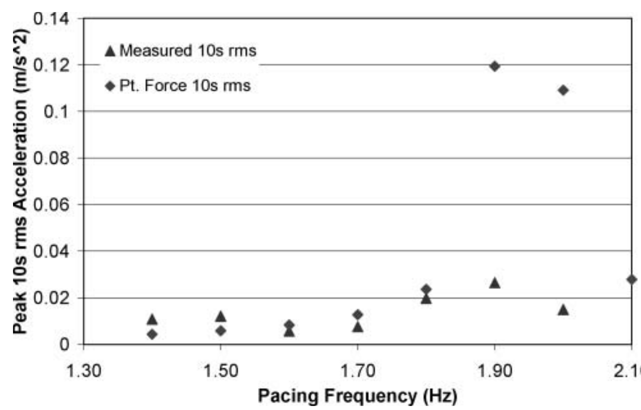

Figure 8. Peak 10 s rms accelerations for TS5—lateral sinusoidal force.

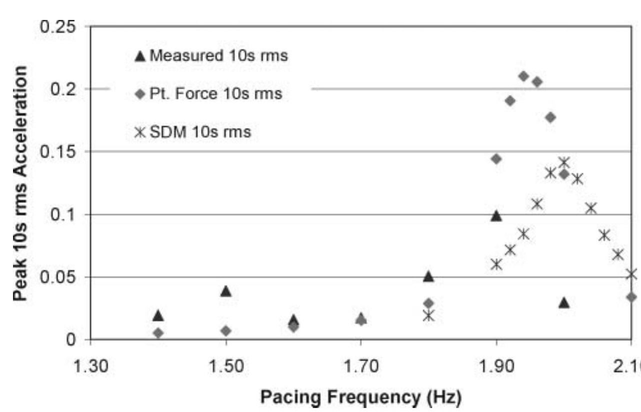

Figure 9. Peak 10 s rms accelerations for TS1—lateral sinusoidal force and spring-damper model.

Table 5. Negative-spring-damper model—the initial and final values for spring stiffness, damping coefficients and foot landing position factor (P_f).

Property	Initial value	Final value								
		TS1	TS2	TS3	TS4	TS5	TS6	TS7	TS8	TS9
Stiffness, k (kN/m)	15	−15	−20	−20	−20	−20	−20	−20	−15	−20
Damping coefficient $(c_v)_1$ (Ns/m)	600	100	600	600	600	600	600	600	600	400
Damping coefficient $(c_v)_2$ (Ns^2/m^2)	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000
P_f	1	1	1	1	1	0.5	1	2	1	2

the measured data set although the magnitude of the shift was a function of, and sensitive to, the exact spring stiffness used.

A feature of all load models, when using a constant lateral load factor of 4% body weight, was that the numerical predictions underestimated the bridge response at low pacing frequencies and overestimated the accelerations at higher pacing frequencies – hence the pacing frequency dependent lateral load factor introduced in equation (5) earlier.

In the case of the negative-spring-damper model the initial and final values for spring stiffness, damping coefficients and foot landing position factor (P_f), used for each test subject, are summarized in table 5. The resulting peak 10 s rms lateral accelerations for TS1 and TS5 are plotted in figures 10 and 11. For both test subjects, across a range of pacing frequencies, better correlation with measured results is achieved using the negative-spring-damper model.

The peak responses predicted by the lateral sinusoidal load model (equation (1)) and the negative-spring-damper model (equation (5)) are also compared with average measured values in table 6. Across the range of test subjects, closer correlation with measured results is achieved using the negative-spring-damper model.

6. Discussion of results

The sinusoidal time-varying point force used to simulate the lateral forces from a single pedestrian is a reasonable representation of the lateral forces generated by persons walking, as measured using force plates on rigid surfaces. This point force model however did not accurately predict the measured bridge response. The peak predicted accelerations occurred when the pacing frequency was 1.96 Hz. This value is exactly twice 0.98 Hz, the first lateral natural frequency of the empty bridge structure. The measured peak response, however, seemed to occur at 1.9 Hz. Numerical analysis, using a validated model, demonstrates that the presence of a person on the bridge is not sufficient alone to cause this level of shift in the natural frequency.

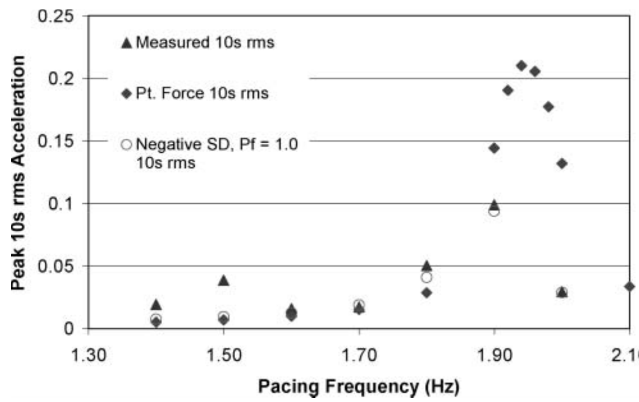

Figure 10. Peak 10 s rms accelerations for TS1.

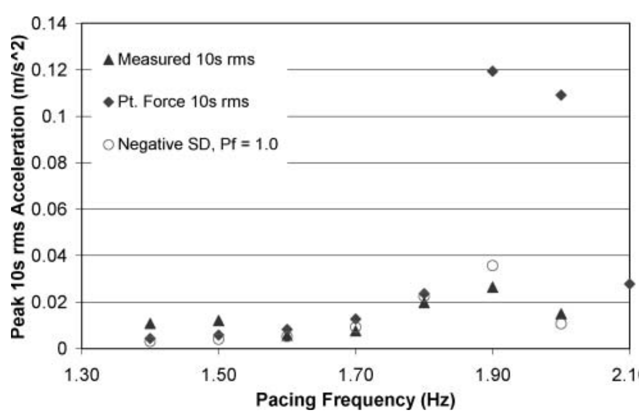

Figure 11. Peak 10 s rms accelerations for TS5.

The optimum correlation between measured and predicted responses was achieved using a spring-damper model with negative spring stiffness. While the use of negative spring stiffness is potentially somewhat controversial its effect is to increase the applied lateral load as a function of the lateral deflection—this is consistent with the response of humans on a moving surface in so far as they tend to feel more comfortable when moving with rather than against

Table 6. Peak 10 s rms lateral accelerations at mid-span (values in parentheses are the percentage error between the predicted and measured).

Test subject	Measured (m/s^2)	Point force $L_F = 0.04$ (m/s^2)	Negative spring-damper $F_L(t) = L_F\, G\sin(\pi f_s t)$ (m/s^2)
TS1	0.099	0.144 (+45.5)	0.094 (−5.1)
TS2	0.070	0.122 (+74.3)	0.072 (+2.9)
TS3	0.040	0.102 (+155.0)	0.065 (+62.5)
TS4	0.069	0.122 (+76.8)	0.072 (+4.3)
TS5	0.026	0.119 (+357.7)	0.036 (+38.5)
TS6	0.0740	0.0972 (+31.4)	0.0575 (−22.3)
TS7	0.08	0.0931 (+16.4)	0.1078 (+34.8)
TS8	0.0666	0.1097 (−35.3)	0.0721 (+8.3)
TS9	0.1205	0.1194 (−0.9)	0.1502 (+24.6)

the structure. The effect of the negative stiffness is also to shift the peak predicted response from 1.96 Hz to 1.9 Hz as measured.

7. Conclusions

A novel lateral load model for single pedestrians traversing footbridges is proposed. The proposed model makes use of a negative spring stiffness to develop increasing lateral loads depending on bridge deflections. Whilst accepting that this manner of load representation may be potentially controversial the resulting load effect is consistent with the expected response of pedestrians on laterally moving footbridges.

Further features of the load model include a pacing frequency dependent load factor and an allowance for the accepted variance in gait between different individuals in the form of a foot landing factor.

Finally, the novel load model is demonstrated to give good, and significantly improved, agreement with measured lateral accelerations, for a range of test subjects, over a range of pacing frequencies, on a full-scale bridge.

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References

Arampatzis, A., Bruggemann, G. and Metzler, V., The effect of speed on leg stiffness and joint kinetics in human running. *J. Biomech*, 1999, **32**, 1349–1353.

Archbold, P., Interactive load models for pedestrian footbridges. PhD thesis, Department of Civil Engineering, University College Dublin, Ireland, 2004.

Bachmann, H. and Ammann, Vibrations in structures induced by man and machines. International Association for Bridge and Structural Engineering, Structural Engineering Document, Zurich, 1987.

Dallard, P., Fitzpatrick, T., Flint, A., Low, A. and Ridsill Smith, R., The Millennium Bridge, London: problems and solutions. *Struct. Eng.*, 2001, **79**, 15–17.

Fanning, P.J., Archbold, P. and Pavic, A., A novel interactive pedestrian load model for flexible footbridges, *Proceedings of the 2005 SEM Conference and Exhibition on Experimental and Applied Mathematics*, Society for Experimental Mechanics, 7–9 June, 2005.

Farley, C. and Gonzalez, O., Leg stiffness and stride frequency in human running. *J. Biomech.*, 1996, **29**, 181–186.

Ferris, D., Liang, K. and Farley, C. (1999) Runners adjust leg stiffness for their first step on a new running surface. *J. Biomech.*, 1999, **32**, 787–794.

Fitzpatrick, T., (2001) *Linking London: The Millennium Bridge*, 2001 (Royal Academy of Engineering: UK).

Fujino, Y., Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge. *J. Earthquake Eng. Struct. Dyn.*, 1993, **22**, 741–758.

Gard, S. and Childress, D., What determines the vertical displacement of the body during normal walking? *J. Prosthet. Orthot.*, 2001, **13**, 64.

Menz, H., Lord, S. and Fitzpatrick, R., Acceleration patterns of the head and pelvis when walking on level and irregular surfaces. *J. Gait Posture*, 2003, **18**, 35–46.

Pavic, A., Reynolds, P., Cooper, P. and Harvey, W.J., Dynamic testing and analysis of Aberfeldy Footbridge. Report by Department of Civil & Structural Engineering, Vibration Engineering Research Section, The University of Sheffield, UK, 2000.

Petersen, C., Theory of Random Vibration and Applications, Work Report on Safety Theory of Structures No. 2/72, Struct. Eng. Laboratory, Technical University Munich, November, 1972.

Simpson, K. and Jiang, P., Foot landing position during gait influences ground reaction forces. *J. Clin. Biomech.*, 1999, **14**, 396–402.

Tilden, C.J., Kinetic effects of crowds. *ASCE Proceedings*, 1913, **39**, 325–340.

Zhang L., Dali X., Mohsen, M. and Fang, L., Stiffness and viscous damping of the human leg, 2000. *Proceedings of the 24th Annual Meeting American Society of Biomechanics*, University of Illinois at Chicago, 19–22 July, 2000.