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Vertical Crowd Load Models for Vibration Serviceability Assessment of Footbridges

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Abstract

Most pedestrian load models prescribed in existing design codes of practice do not consider any interaction between the pedestrian and the moving structure. However, it is now widely accepted that this interaction is critical to the forces exerted on the bridge and thus the bridge response to such forces. Several new approaches to modelling crowd loading, including some stochastic methods have been proposed in recent times. Nonetheless, these do not fully simulate the effect of the interaction between the pedestrian and the structure on the load pattern exerted by the pedestrian.

This paper examines some of these approaches and presents an innovative technique for modelling vertical crowd forces. This deterministic load model may provide the basis for a meaningful probabilistic approach to the issue of footbridge vibration serviceability.

Keywords: pedestrian loading, crowd synchronisation, human-structure interaction, footbridge, vibration serviceability.

1 Introduction

Improvements in the performance of structural materials and the growing influence of more daring architectural designs have led to lighter and more slender footbridges in recent times. An associated feature of these lighter structures is that they tend to be more susceptible to dynamic excitation from various sources, including pedestrian induced forces. Some high-profile examples of recent bridges, which have experienced noticeable responses to dynamic pedestrian induced loads are the Millennium Bridge London [1], the Pont du Solferino, Paris [2] and the T-Bridge, Japan [3]. Each of these exhibited lateral sway under crowd loading which resulted in feelings of discomfort to the bridge users. Consequently, much of the latest research into vibration serviceability performance of footbridges has concentrated on the lateral excitation, with less focus on the vertical response.

Nonetheless, pedestrian walking produces major reaction force components on a structure in three orthogonal directions – vertical, longitudinal (or saggital) and lateral (or medio-lateral). Each of these forces contributes to the response of the structure. In order to understand the source of these forces, one must examine the phenomenon of human walking. Much of this work has been done in the field of biomechanics.

Although human locomotion is a very complex process, Davis et al [4] describe walking or "gait" as a cyclic activity for which certain discrete events have been defined as significant. The following characteristics can be used to describe human walking: pacing frequency, stride length and step width.

The pacing frequency refers to the inverse of the time taken from the initial contact of the left foot with the ground to the initial contact of the right foot immediately thereafter and corresponds to the rate of application of vertical forces. Reported values of normal pacing frequencies indicate that the average pacing rate is between 1.6Hz and 2.4Hz [5].

Research has shown a relationship between stride length, pacing frequency and the height of the pedestrian [6,7]. More recent work by Barela et al [8] reports mean values for stride length recorded from 10 adults walking on land as 0.66m, while for 10 elderly adults, this further reduced to 0.58m. Lai et al [9] reported a difference in the relationship between stride length and pedestrian height between obese people and non-obese, with those overweight having a smaller stride length to height ratio. Clearly there are a huge range of factors affecting normal walking stride length, but many design approaches assume a value of 0.90m, which appears to be slightly longer than average reported values.

Donelan et al [10] measured the preferred step width in normal walking by measuring the mechanical and metabolic costs associated with manipulated lateral foot placements. They found a coupling between stride length and step width and reported that, based on the minimum metabolic cost involved in the action, the step width was approximately 12% of the stride length, while Bauby & Kuo [11] report a value of 13%. Cho et al [12] report that female adults displayed smaller step widths than their male counterparts in a series of gait analysis tests, however, comparison of their step width to stride length ratio reveals a ratio of approximately 14% for both male and female participants. Again, there are a number of factors affecting step width.

Each of the above characteristics of human walking is important in assessing the reaction forces imparted from a pedestrian to a structure. While many factors affect each of these characteristics and there is a level of variability associated with each trait, some simplifying assumptions and generalisations are made in deriving numerical models of pedestrian loading. Traditional pedestrian load models make use of some of these assumptions and the accompanying results from fixed force plate tests.

This paper presents some of the load models currently proposed for predicting the vertical structural response of footbridges due to pedestrian loading, which require not only pedestrian load factors, but which also feature interaction characteristics both between the individual pedestrians and the vibrating structure and also interaction between a number of pedestrians in a crowd situation. This paper addresses the issue of vertical vibration only.

2 Vertical Force Models for Individual Pedestrians

Design codes of practice differ in their approach to the issue of potential structural excitation due to pedestrian loads. Some codes [13, 14] adopt a frequency tuning method whereby, natural frequencies which are likely to be excited by pedestrian loading are prohibited. This appears to be a very conservative and restrictive approach however, limiting the potential options available to bridge designers. Moreover, this method does not consider the effects of mass on the vibration performance of the bridge [15].

Other codes such as BS5400 and the Eurocodes prescribe limits on the accelerations due to individual pedestrian load models, generally derived from results of walking on fixed force plates and thus not considering any interaction with the structure upon which these loads are acting.

2.1 Moving Point Load Model

BS 5400 adopts a simple load specification derived by Blanchard et al [16] having the form of a pulsating force of amplitude P moving along the span, with a velocity of 0.9 times the pacing frequency, f_s. Fanning et al [17] showed that a more accurate moving point force model can be used, which consists of a moving point load F(t) prescribed by:

$$F(t) = G + r_1 G\sin(2\pi f_s t) \qquad (1)$$

where:

F(t) is the forcing function

G is the weight of the person crossing

r_1 is the dynamic force component factor, which is dependent on the actual pacing frequency

f_s is the pacing frequency (Hz)

t is the time of load application

This represents a cyclical sine wave load model, which is moved across the span of the bridge. However, this model does not take the mass of the person walking into account and tends to be conservative in its prediction of vertical accelerations.

2.2 Moving Mass and Dynamic Component Model

Fanning et al [17] further refined this model to include the mass of the pedestrian and a dynamic component. However, the authors showed that, while this improved the accuracy of the predicted mid-span accelerations from a single person traversing a footbridge, it also tended to overestimate the actual measured vertical response. Figueiredo [18] incorporated a factor into a similar model to account for the force exerted by the heel strike on the bridge as proposed by Varela [19], with no significant improvement on the predicted mid-span vibration levels.

2.3 Spring Mass Damper Interactive Model

Neither of the above load models accounted for any interaction between the crossing pedestrian and the vibrating structure. Such interaction has been shown to reduce the actual applied vertical walking force when walking on a flexible structure. For example, Racic [20] reports observations by Baumann and Bachmann [21] that vertical walking load was up to 10 percent higher if measured on stiff ground in comparison to measurements on a flexible pre-stressed concrete beam. Similarly, Pimentel [22] showed that in the case of flexible footbridges walking forces were considerably lower than those previously reported on solid surfaces. In an attempt to address this, Archbold [23] proposed a model whereby the person was modelled as a vertical spring-mass-damper single degree of freedom system, as shown in Figure 1. This model was given a stiffness which is similar to reported leg stiffness values and the mass of the pedestrian was applied and subjected to a dynamic force component as above. This model provided extremely accurate predictions of mid-span accelerations in the case of a footbridge which was subjected to 100 individual pedestrian crossings.

Each of the models above accounts for the loads applied by a single pedestrian only. The loads applied by a crowd are believed to be derived from the superposition of a number of individual load models. Blanchard et al [16] however considered that the issue of crowds of people walking in step was not of importance as they believed that people in such crowds would "break step" upon perception of vibrations in the structure upon which they were walking. However, more recent work has attempted to address the phenomenon of crowd synchronisation i.e. people walking at the same pacing frequency, sometimes triggered by resonance of one of the structure's natural frequencies.

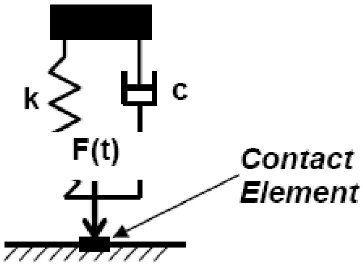


Figure 1. SDOF Spring Mass Damper in Contact with the Bridge Deck

3 Synchronisation of Vertical Crowd Forces

Synchronisation among crowds walking on a lively platform is dependent on two factors. The first factor is the synchronisation which is likely to occur between people walking in a crowd situation, regardless of any contribution from the structure. This is more likely to increase with crowd density but can also depend on factors such as walking speed, pacing frequency and a stochastic value of synchronisation probability. The second contributing factor is the likelihood of persons "locking in" with bridge deck movements. This is affected by the amplitude and frequency of the vertical movements of the structure [24].

While the lock-in effect is treated through the above spring mass damper model, the effects of people synchronizing with other people in a crowd has not yet been dealt with. This phenomenon has, however, been investigated by several authors who proposed models for the response due to synchronization of people within a crowd. The general approach has been to superimpose the effects of one person for a number of people. The number of people deemed to be "in-step" with each other has been determined in a number of ways.

Bachmann and Ammann [25] report that a mathematical description of this synchronization phenomenon would be difficult but propose an enhancement factor, m, to be applied to the force from a single pedestrian based on the Poisson distribution of the arrival probability for pedestrians as reported in Matsumoto et al [26]:

$$m = \sqrt{\lambda T_0} \tag{2}$$

where:

m = enhancement factor to be applied to the vibration amplitude caused by a single pedestrian

λ = mean flow rate (persons per second over width of deck) for a certain period of time ($\lambda_{max} \approx 1.5$persons/s m)

T_o = time necessary to cross bridge of length L at speed v_s,i.e. $T_o = L/v_s$

$\lambda\ T_o$ = number of persons simultaneously on the bridge at the given mean flow rate and is denoted as N. Therefore the enhancement factor proposed by Bachmann and Ammann [25] based on work by Matsumoto [26] can be re-written as:

$$m_B = \sqrt{N} \tag{3}$$

Grundmann et al [27], meanwhile defined the probability of synchronisation as being dependent on the amplitude of the structure's acceleration. Rearrangement of their formula for single span bridges gives an enhancement factor, m_G of:

$$m_G = 0.135N \tag{4}$$

Fujino et al [3] reported on investigations into the excitation of the T-bridge in Japan under crowd loading. They reported approximately 20% of the crowd were synchronised. This could be represented by:

$$m_F = 0.2N \tag{5}$$

More recent work by Nakamura et al [28] supports this value of 20% synchronisation, with the level rising to almost 50% when the frequency of lateral vibration was close to 1.0Hz. Figure 2 shows how these three methods for calculating the enhancement factor compare for different numbers of pedestrians on the bridge.

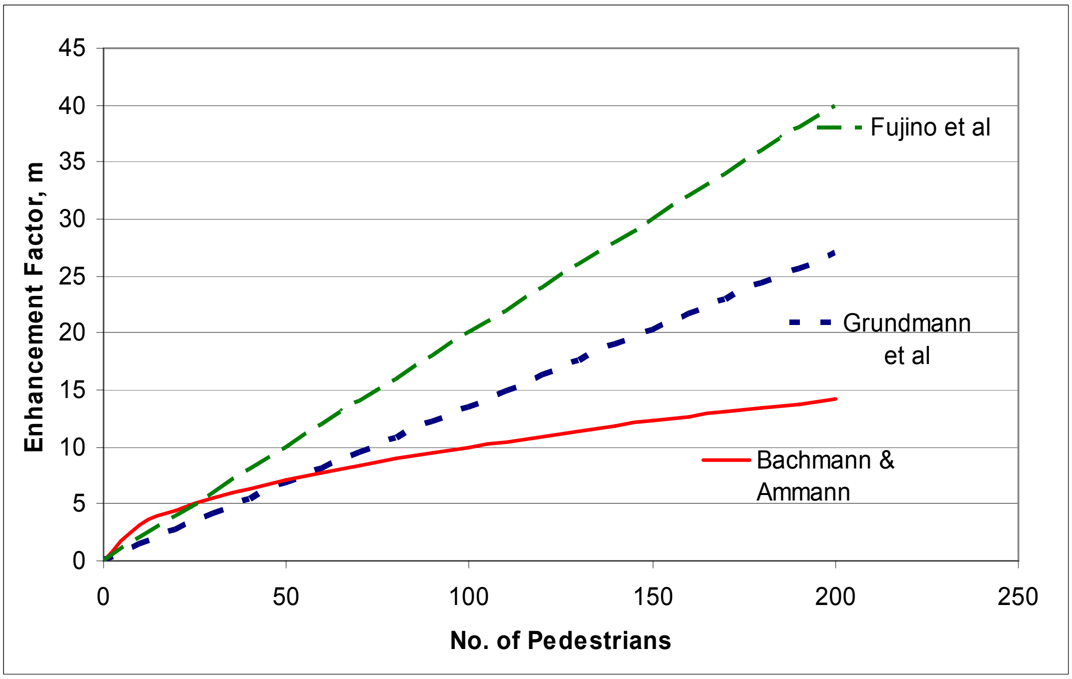
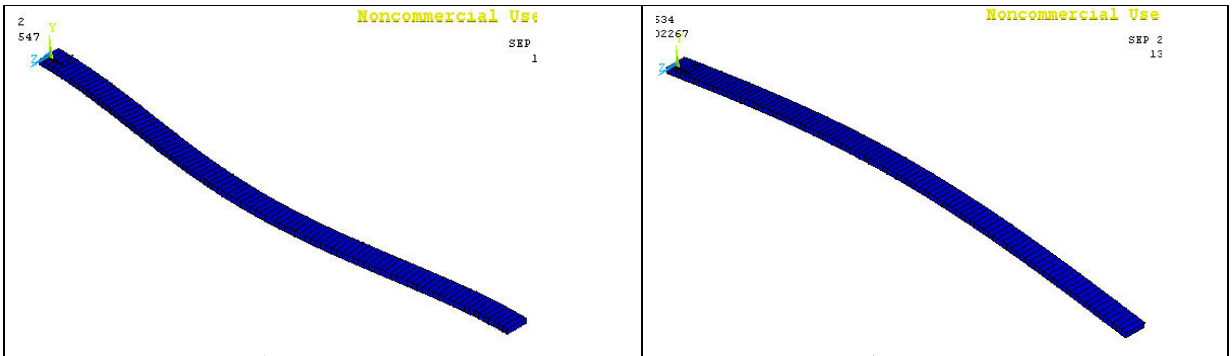


Figure 2. Proposed Enhancement Factors for Vertical Loading Due to Crowd Synchronisation

It is noticeable that none of the above models have been tested on other structures. Other researchers have attempted to use other techniques for estimating the force functions due to crowd synchronisation using techniques such as computational flow dynamics and methods originating from wind engineering but the success of any such methods has been limited.

4 Modelling Programme

The focus of this paper is to evaluate the models discussed above in terms of predicted acceleration at mid-span for a single span footbridge. Also, an innovative approach, which considers interaction between the pedestrians and the moving structure is introduced. In order to do this a finite element model of a typical bridge was developed using ANSYS V11.0. This finite element model was developed such that the bridge had a fundamental vertical natural frequency in the region of general pacing frequencies and a fundamental lateral natural frequency of approximately 1Hz. Thus, this bridge would be susceptible to excitation by crossing pedestrians. The bridge was assigned a deck width of 2m and had an overall span of 50m. Modal analysis of the FE model revealed an actual lateral natural frequency of 1.03Hz and a first vertical natural frequency of 2.38Hz. The first two mode shapes are shown in Figure 3 below.



(a) 1st Lateral Mode Shape
Natural Frequency 1.03Hz

(b) 1st Vertical Mode Shape
Natural Frequency 2.38Hz

Figure 3. First two mode shapes of FE model

The finite element model was subjected to loading from an individual pedestrian using each of the three techniques described in Section 2. The maximum vertical acceleration due to each of the techniques was predicted to be 0.061m/s^2. There appeared to be negligible difference between the calculated response from each method, which is most likely due to the rather high self-weight of the bridge.

The effects of vertical forces from crowd synchronisation were then modelled using 4 methods. The first three methods were as described in Section 3 of this paper and reported from the literature, whereby enhancement factors were applied to the predicted response for one person from the point force model, based on an anticipated level of crowd synchronisation.

The fourth method is a novel technique, proposed by Archbold [29], which employed the use of the spring-mass-damper model for modelling a single pedestrian. This spring-mass-damper model was then used to individually model each pedestrian present on the bridge for two crowd densities of 0.75 persons/m^2 and 1.5 persons/m^2 respectively. Dynamic force components were applied to those deemed to be “in-step”, assuming an even distribution over the entire bridge deck, while those not “in-step” were assumed to be randomly distributed with no dynamic load component applied. An initial synchronisation level of 20% was used. The effect of varying this level of synchronisation among the crowd on the overall bridge response was then also investigated. These methods are denoted as B, G, F and A for the synchronisation models proposed respectively by Bachmann et al (1987), Grundmann et al (1993), Fujino et al (1993) and Archbold[29]. The suffix 75 represents a crowd density of 0.75 persons/m^2, while the 150 represents a crowd density of 1.5 persons/m^2.

5 Results

The results from the models employed are examined in terms of the maximum predicted 5s root mean square (rms) vertical acceleration at mid-span of the bridge. Figure 4 shows the maximum 5s rms vertical acceleration at mid-span for each of the four models over the normal range of pacing frequencies at a crowd density of 1.5 persons/m^2. It can be seen that the critical frequency is 2.3Hz, as expected from the natural frequency of the first vertical mode shape of the bridge model. Away from this frequency, the accelerations are not noticeable. As a result, the predicted accelerations at a pacing frequency of 2.3Hz are examined in further detail. The peak values at 2.3Hz vary considerably from 0.75m/s^2 from the Bachmann et al enhancement factor to 1.84m/s^2 from the Fujino et al model. Both exceed the limit prescribed in the Eurocodes of 0.7m/s^2.

Figure 5 shows the values of the peak 5s rms accelerations at mid-span for a pacing frequency of 2.3Hz and two crowd densities of 0.75 persons/m^2 and 1.5 persons/m^2 respectively. As expected, the results based on both the Fujino et al (1993) and the Grundmann et al (1993) models show a linear relationship, while the results from the Bachmann et al (1987) model are proportional to the square root of the number of people present. The model proposed, herein however, predicts an increase from 0.38m/s^2 to 0.98m/s^2, which is a 157% increase based on a 100% increase in the number of people on the bridge. This implies that the relationship between the actual response and the number of people present on the structure is not, in fact, linear. This fits well with anecdotal evidence that the more a structure vibrates, the more sensitive people are to those vibrations, which leads to them walking “in-synch” with the structure thus increasing the applied force. This appears to capture one of the elements of crowd-structure interaction. It is notable that the predicted response from this model at 1.5 persons/m^2 lies within the range of values predicted from the other methods, while it is slightly lower than the other methods at a density of 0.75 persons/m^2.

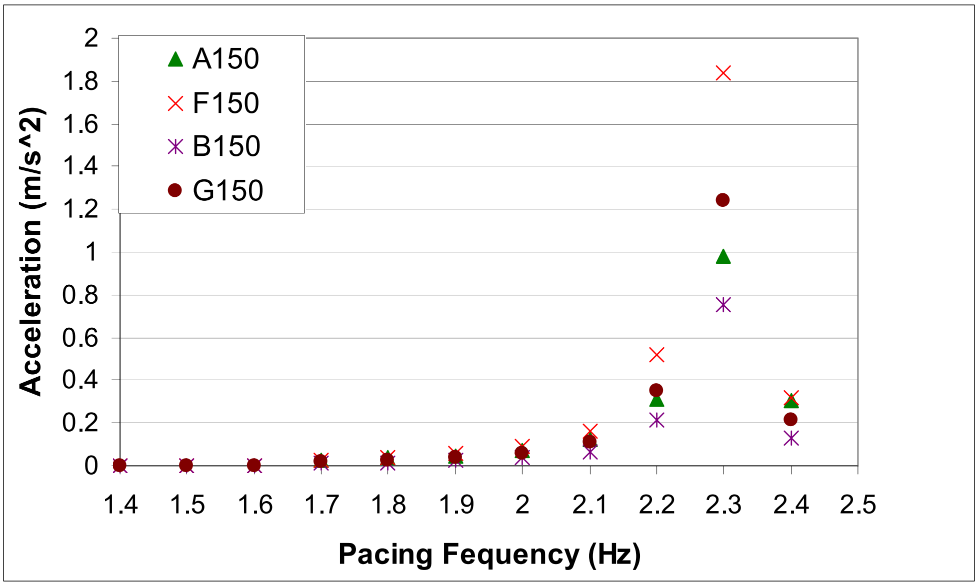


Figure 4. Maximum 5s rms vertical accelerations at a range of pacing frequencies

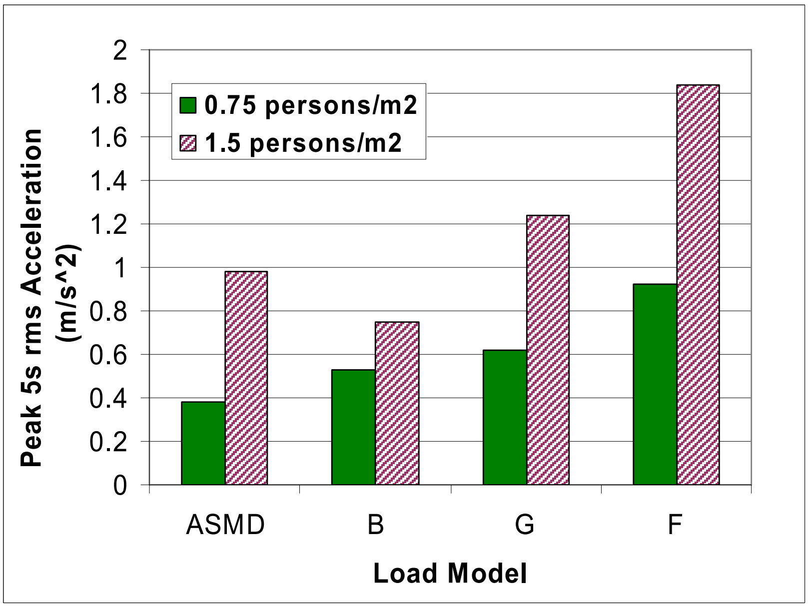


Figure 5. Peak 5s rms accelerations at a pacing frequency of 2.3Hz for two crowd densities

The above results were based on a crowd synchronisation of 20% applied to the individual spring-mass-damper models of the pedestrians. Figure 6 shows the results yielded from similar models where the crowd synchronisation is varied between 5% and 50%. Again, this illustrates that the relationship is not simply characterised by a linear increase in the predicted response from a similar increase in the level of synchronisation within the crowd. It should be noted that none of the models examined in this paper deal with the issue of a threshold of vibration which leads to crowd lock-in with the structure. This is an issue worthy of further investigation.

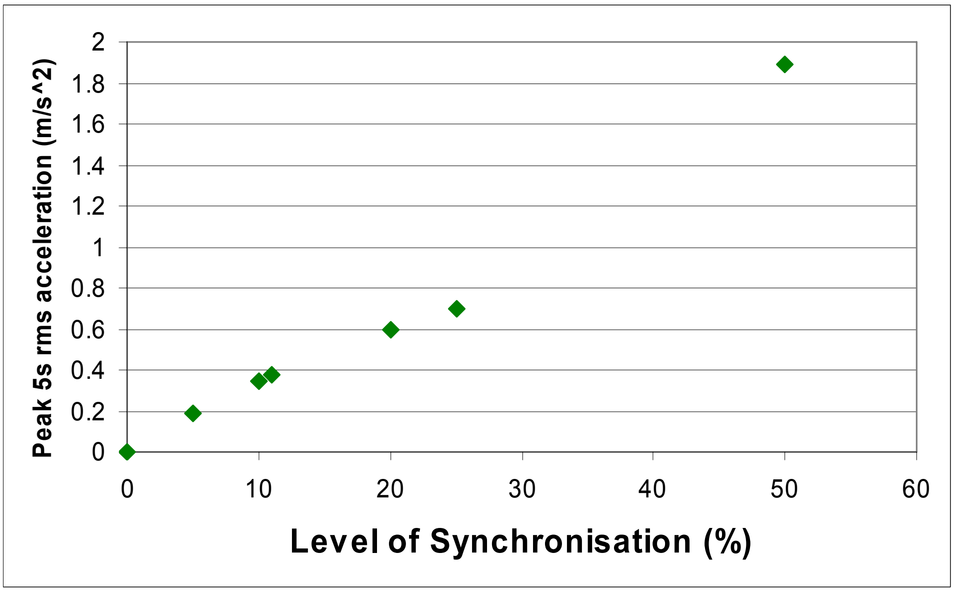


Figure 6. Peak 5s rms accelerations at a pacing frequency of 2.3Hz for varying levels of crowd synchronisation

6 Conclusions

In this paper several techniques for simulating the vertical forces caused by crowds of pedestrians walking across a flexible footbridge were examined. Approaches adopted in international codes of practice for dealing with vibrations due to crossing pedestrians were described, along with a number of models used to represent individual pedestrians.

The phenomenon of crowd-structure interaction is essentially composed of two elements – the interaction between the crowd members and a moving surface and also the interaction between pedestrians within a crowd. This paper addresses both of these issues.

Approaches reported in the literature for dealing with this are presented and evaluated for a test 50m long test structure with a vertical natural frequency in the range susceptible to excitation from walking pedestrians.

A new approach, whereby a spring-mass-damper model, which has been shown to accurately represent the response from an individual pedestrian, is employed to model each of the individual pedestrians present on the test bridge. This model is examined for the effects of different crowd densities and a range of levels of synchronisation within the crowd.

It should be noted that this method of predicting the actual response has not been verified in the field at this point. Neither has the possibility of a threshold value of vibration, which may lead to pedestrians "locking in" with the moving structure. These issues are currently being addressed by the author.

The results do show that the predicted response is not necessarily linearly proportional to the level of crowd synchronisation, as suggested by previous attempts to model this phenomenon. Nonetheless, the level of acceleration predicted for a crowd density of 1.5 persons/m^2 and a synchronization level of 20%, which appear to be realistic values for normal crowd behaviour, are within the range predicted by the earlier approaches. This new model considers both interaction between the individual pedestrian and the moving surface and also pedestrian behaviour within a crowd, which has not been done before. It therefore lends itself well to use as a basis of a stochastic approach to the issue of crowd structure interaction in terms of vertical vibration serviceability of footbridges.

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