Characteristic Vertical Response of a Footbridge Due to Crowd Loading

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ABSTRACT: The characteristic vertical vibration of a flexible footbridge subject to crowd loading is examined in this paper. Typically, bridge vibrations produced from a crowd of pedestrians are estimated by using an enhancement factor applied to the effect caused by a single pedestrian. In this paper, a single pedestrian model, represented by a spring mass damper, which incorporates variables such as pedestrian mass and body stiffness, is used to calibrate a computationally efficient moving force model. This calibrated moving force model is further used in Monte Carlo simulations of non-homogenous crowds to estimate characteristic vertical vibration levels. Enhancement factors, which could be applied to simple single pedestrian moving force models in estimating the response due to a crowd are thus derived. Such enhancement factors are then compared to previously published values. It is found that the greatest difference between the spring mass damper and moving force models respectively occurs when the bridge frequency is at the mean crowd pacing frequency. For bridges with frequencies even slightly removed from this mean, moving force models appear adequate.

KEY WORDS: Footbridge; vibration; vertical; crowd; pedestrian; characteristic; Monte Carlo; Enhancement Factor.

1 INTRODUCTION

1.1 Background

The issue of excessive vibrations of footbridges due to pedestrian loading has been well documented in the past decade. Typically, bridge vibrations produced from a crowd of pedestrians are estimated by using an enhancement factor applied to the effect caused by a single pedestrian represented by a moving force. However, there are deficiencies in the load models used to predict these responses to individuals. The models are commonly deterministic, and moreover do not consider interaction between the bridge and pedestrian.

The need for a probabilistic approach to pedestrian loading has been acknowledged for a long time [1], [2]. The models used in design codes such as BS 5400 [3], [4] and Eurocode 5 [5] use deterministic moving force models to predict the response of a single pedestrian. These models are commonly unable to accurately predict the response due to a single pedestrian and usually overestimate it significantly [6]. This has resulted in difficulties applying universal enhancement factors to such responses.

It was reported by Archbold [7] that a moving force model may also be conservative in its predictions as it does not consider interaction between the pedestrian and the moving bridge surface. This can lead to overestimation of the acceleration response due to both single pedestrians and crowd loading. Serviceability assessment of bridge structures using these models could therefore be overly conservative.

1.2 Approach of this work

In this paper a moving spring mass damper (SMD) model is developed to represent a single pedestrian. The single degree of freedom SMD accounts for leg stiffness and damping, and facilitates consideration of some of the interaction between the pedestrian and the bridge. Biomechanics literature was reviewed to identify suitable mechanical properties, primarily the spring stiffness in the SMD, for the pedestrian. The bridge used in the model is a simply-supported beam, chosen to be susceptible to excitation from typical pedestrian pacing rates. To model the footfall force, a time-varying harmonic force is applied to the pedestrian mass.

Using this single-pedestrian model, a crowd loading model is developed. The crowd model uses statistical distributions of pedestrian parameters to derive characteristic responses, for various synchronization levels and crowd densities.

The following distributions are used to represent the variations in the characteristics of the pedestrians on the bridge: pedestrian weight is represented by a log-normal distribution, while stride length, pacing frequency and leg stiffness respectively are represented by a normal distribution. The phase angle of those not synchronized is assigned a uniformly random distribution. Pedestrians' starting locations are based on a Poisson arrival process and are thus given gaps described by the exponential distribution.

The proportion of pedestrians taken to be synchronized, that is, walking in step with each other, is termed the level of synchronization. Synchronized pedestrians are assigned the same pacing frequency (randomly chosen from the population normal distribution) and phase angle (again, randomly chosen). The levels of synchronization of the pedestrians on the bridge are chosen to allow comparison with reported values as will be discussed in Section 5.1.

1.3 Relationship to design codes

Design codes for pedestrian bridge excitation have traditionally used a moving force model to represent pedestrians [6] to which an enhancement factor may be applied to determine the design response due to crowds. This work determines suitable enhancement factors, based on improved pedestrian and crowd modelling, and characteristic responses thereof, to arrive at improved enhancement factors that may be applied to the simpler moving force model results. In this way, this work should aid designers who are using moving force models of pedestrians to arrive at less conservative estimations of vertical acceleration response to crowd loading.

2 PEDESTRIAN STIFFNESS AND DAMPING

2.1 Use of SMD models to represent human loading

In repetitive physical activity, such as running, hopping and trotting, a subject bounces on the ground in a spring-like manner [8]. As a result a number of authors have represented the leg, while running, as a spring-mass model [9]. Geyer et al [10] state that walking too, is a bouncing gait.

Rapoport et al [8], however, stated that physiologically the concept of constant mechanical stiffness may not be applicable and so human joints are not simple mechanical springs. They report that joint stiffness is nonlinear in nature as damping may be present and as a result a model which accounts for this damping may improve the model predictions.

Lee and Farley [9] highlighted that spring and damping elements have been incorporated into the legs of some models of walking in order to match ground reaction force (GRF) patterns observed in human walking. They report that the values used in these models are generally higher (k_P = 12-35.5 kN/m) than the leg stiffness values reported for normal walking ($k_{leg} \approx$ 11 kN/m).

2.2 Hopping

Zhang et al [11] carried out an analysis on three tests subject standing on a force platform from which leg values were obtained. In the tests, the subject's centre of mass (COM) was moved up and down in a small amplitude and random pattern by lifting the trunk through a harness system which flexed the knees, ankles and hips slightly. The subject's feet remained stationary on the platform and markers were placed at different points on their body to track the movement. Following their research of a subject bouncing in the one position, they reported a leg stiffness value of 28.5 kN/m and a damping value of 950 Ns/m, which equates to a damping ratio of 0.3 for a mass of 78 kg.

Rapoport et al [8] found the leg stiffness of eight female subjects with an average body mass 54.8 kg to be values of 9.8, 14.6 and 20.9 kN/m respectively when hopping up and down at frequencies of 1.53, 1.87 and 2.20 Hz, respectively. It is evident that the stiffness values increase with hopping frequency. This supports the theory the joint stiffness increases proportionally with increasing impact frequency, due to the reduced stance time, leading to an increase in the overall leg stiffness.

Lebiedowska et al [12] obtained leg stiffness values by getting eight test subjects to hop off a block of wood (height of 14 cm) and land on their heel first, then onto their flat foot and finally onto the ball of their foot (toes). The stiffness values obtained are as follows: 61.23 ± 21.52 kN/m (damping ratio 0.26), 56.25 ± 15.28 kN/m (damping ratio 0.24, and 29.77 ± 12.05 kN/m (damping ratio 0.26) for landing on their heel, flat foot and toes, respectively. During this test, the subjects were asked to keep their knee in the locked position and so, as stated by the authors, these stiffness values are significantly higher than those expected for walking, as knee flexion is commonly present throughout the gait cycle.

2.3 Walking

Bertos et al [13] incorporated a shock absorber into a rocker based inverted pendulum model to cater for the 'viscoelastic properties of muscles, neuromuscular feedback and geometry changes of joints in the leg while walking'. This resulted in a reduction in the movement of the COM. They used a test subject with a mass of 95 kg walking at a velocity ranging from 0.8 to 2.2 m/s. From this the authors estimated the leg stiffness and damping ratio so their model would match the measured results. They obtained a vertical displacement of the COM against time fit of 75-80% for slow speeds and 90-95% for normal and fast speeds. From this, graphs of walking speed against damping ratio and stiffness were developed. The damping ratio ranged between 0.40 and 0.70 across the velocity range and the stiffness from 2 kN/m at 0.75 m/s up to 13 kN/m at 2 m/s. These stiffness values are significantly lower than those quoted by Lee and Farley [9].

Gayer et al [10] stated that the inverted pendulum cannot reproduce the characteristic M-shaped GRF and so does not represent the stance phase of a pedestrian correctly. Also Lee and Farley [9] found that the inverted pendulum cannot reproduce accurately the trajectory of the COM as it overestimates its height at mid-stance. As a result, since Bertos et al [13] used the trajectory of the standard inverted pendulum as their input for their new model, the input may be overestimated thus requiring excessive amounts of damping to match the displacement of the test subject's COM.

Gayer et al [10] used a bipedal spring mass model to represent five test subjects walking. The point mass was placed on two massless spring elements. They investigated the angle of attack, made with the leg and the ground before touchdown. They reported an increase in stiffness with an increase in attack angle from 14 kN/m at 69° to 20 kN/m at 76°.

2.4 Running

Numerous authors have published values of leg stiffness for humans whilst running. Arampatizis et al [14] give extensive coverage of previous research in this area. They highlighted that previous authors reported different findings from their published research. Some reports argue that the stiffness of the leg is not dependent on velocity while others suggested that it is.

Following their own research, Arampatizis et al [14] concluded that the stiffness of the leg does in fact increase with an increase in running velocity. The authors recorded force plate measurements from thirteen test subjects (mass: 80.68 ± 4.99 kg) running across a force plate platform at varying velocities from 2.5 to 6.5 m/s. The running was videoed using two high speed cameras and reflective markers were placed on the joints of the test subjects to improve the quality of the video analysis. A spring-mass model was created to mimic that of the recorded data. The leg stiffness (k_{leg}) and the effective vertical spring stiffness (k_P) were calculated. These values were obtained by dividing the ground reaction force by the change in the length of the spring mass model and the vertical length change of the subject's centre of mass (COM), respectively. The k_{leg} values obtained were between 25.29 and 35.21 kN/m at velocities of 2.61 to 6.59 m/s. The k_P values obtained were between 25 and 92 kN/m at the same velocities, this takes into consideration part of the torso. It was reported that this increase in overall stiffness with the increase in velocity is mainly due to the increase in stiffness of the knee joint. It was acknowledged by the authors that this value was higher than other authors had predicted.

Ferris et al [15] found that humans also adjusted their leg stiffness to accommodate changes in surface stiffness, thus allowing them to maintain similar running mechanics on different running surfaces. If humans were not to adjust their leg stiffness, their ground contact time and COM displacement would increase as surface stiffness decreases. In their research they used five test subjects running on surfaces with different stiffness values. The subject's leg stiffness values showed a reduction in stiffness with the increase in stiffness of the walking surface. One subject's values ranged from 16 kN/m on a surface with a stiffness of 15 kN/m to 12 kN/m on a surface stiffness of 34 kN/m. The researchers found that vertical stiffness values of the test subjects ranged from 26 to 35 kN/m but did not vary with surface stiffness. Thus a human changes their leg stiffness in order to maintain their COM in a similar position while running regardless of surface stiffness. This is similar to that found by Arampatizis et al [14].

3 PEDESTRIAN AND BRIDGE MODELLING

3.1 *Bridges and bridge models*

The bridge considered is a simply-supported, 50 m long beam with a mass of 500 kg/m and width of 2 m. The flexural rigidity was altered between simulations to achieve the range of natural frequencies considered in this study. Damping is taken to be 0.5% for the first two modes, with Rayleigh damping assumed thereafter. It is acknowledged that this will dampen the influence of higher modes.

For this work, the bridges examined are modelled in two ways, depending on the purpose. Modal analysis is used for both single pedestrian moving force and single pedestrian spring-mass-damper models. In both cases, 5 modes are used to estimate the bridge response.

For the crowd loading simulations, a finite element model of the bridge was used to estimate the response. The beam was modelled using 10 beam elements, with lumped mass assumed. Transient solutions are obtained using the Newmark-β method. Each pedestrian is described by a moving force which varies with time, as will be later discussed. Each moving force is distributed to the adjacent nodes according to the beam element shape functions as described in Wu et al [16]. The forces on the bridge due to the crowd at any point in time are taken as the superposition of the individual pedestrian forces.

The response of interest in this study is taken as the vertical midspan acceleration. The vibration response is assessed using a 5-second root-mean-square (RMS) moving average from the acceleration history of each simulation [7]. The maximum of the RMS from any one particular scenario is taken as the response of the bridge to that particular loading scenario [17].

3.2 *Single pedestrian Moving Force (MF) model*

While walking, the vertical force induced by both feet is assumed to be of the same magnitude and to be periodic [18], [19]. A typical vertical ground reaction force (GRF) produced from walking is presented in Figure 1. This shows a peak from the heal striking the walking surface and toe push-off in addition to a trough when the foot is flat on the ground, mid stance. During walking, one foot is always in contact with the walking surface and as a result the GRF traces from consecutive footfalls partially overlap in time. The GRF can be represented as a Fourier series and some authors have used many terms to model the GRF with good accuracy [20], [21]. However, for this work, the walking force is taken as just the first harmonic of the Fourier series, and is thus given by the sine wave approximation shown in Figure 1.

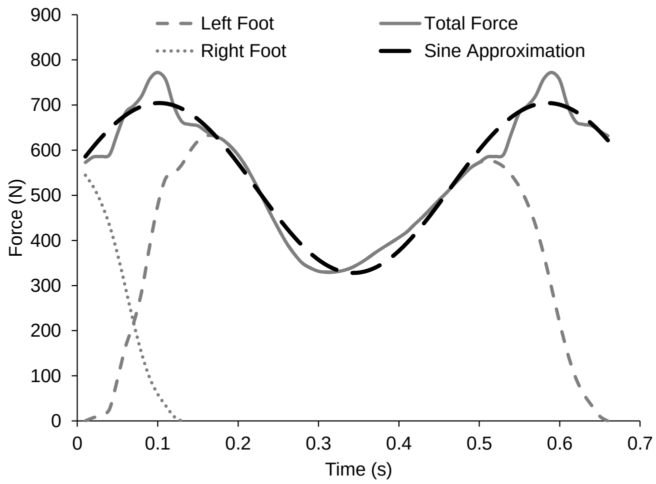


Figure 1. Typical vertical ground reaction force and approximated model force.

From Figure 1 a single pedestrian action is considered to be described according to:

$$P(t) = m_P g\left[1 + r\sin\left(2\pi f_p t\right)\right] \tag{1}$$

In which, m_P is the pedestrian mass, g is the acceleration due to gravity, f_p is the pacing frequency, and r is the dynamic force component from Fanning et al [22], given by:

$$r = 0.25 f_p - 0.1 \tag{2}$$

Using a modal solution for the above problem (see for example [23], [24]), the force for the nth mode is given by:

$$P_n = \int_0^L p(x,t)\phi_n(x)\,dx \tag{3}$$

The nth mode shape is given by:

$$\phi_n(x) = \sin\frac{n\pi x}{L} \tag{4}$$

And the force is as per Figure 1. Thus Equation 1 becomes:

$$P_n = m_P g\int_0^L\left[1 + r\sin\left(2\pi f_p t\right)\right]\delta\left(x - vt\right)\phi_n(x)\,dx \tag{5}$$

Where $\delta(\cdot)$ is the Dirac delta function, required to locate the load on the beam, shown in Figure 2. Equation 5 evaluates to:

$$P_n = m_P g\left[1 + r\sin\left(2\pi f_p t\right)\right]\sin\frac{n\pi vt}{L} \tag{6}$$

The solution for each of the N modes can be found through summation of the equivalent single-degree-of-freedom model solutions, found for the modal generalized coordinates, q:

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q} + \omega_n^2 = \frac{P_n}{M_n} \tag{7}$$

Where M_n is the modal mass and is $2/mL$ in which m is the mass per metre of the beam of length L; ξ_n and ω_n are the damping ratio and circular natural frequency for mode n.

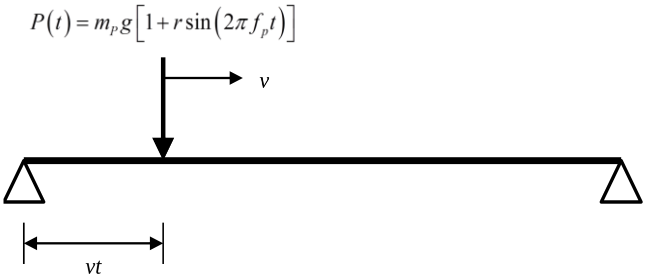


Figure 2. Moving pulsating force model of a pedestrian.

3.3 *Single pedestrian Spring-Mass-Damper (SMD) model*

To better account for the known mechanical properties of pedestrians outlined in Section 2, a spring-mass-damper (SMD) model is adopted, as shown in Figure 3. In this model, the pedestrian mass is supported by a massless spring and damper which represent the stiffness and damping of the human body, (specifically those acting between the centre of gravity and contact surface of the bridge). A pulsating force is also applied to the bridge surface at the pedestrian location, to represent the first harmonic of the walking force function, as shown in Figure 3.

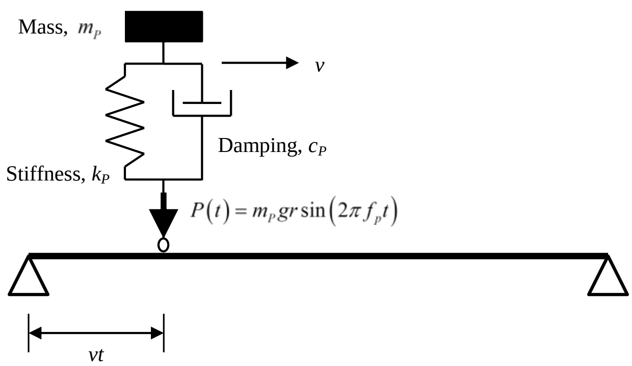


Figure 3. Mixed spring-mass-damper (SMD) and pulsating force model of pedestrian-bridge interaction.

The SMD-pulsating force solution can be expressed in N+1 coupled modal coordinate equations as:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \tag{8}$$

In which,

$$\mathbf{M} = \begin{bmatrix} \mathbf{I}_{N\times N} & \mathbf{M}_{12} \\ \mathbf{0}_{1\times N} & m_P \end{bmatrix} \tag{9}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{0}_{1\times N} \\ \mathbf{C}_{21} & c_P \end{bmatrix} \tag{10}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{0}_{1\times N} \\ \mathbf{K}_{21} & k_P \end{bmatrix} \tag{11}$$

$$\mathbf{Q} = \begin{Bmatrix} \mathbf{Q}_B \\ 0 \end{Bmatrix} \tag{12}$$

$$\mathbf{M}_{12} = \frac{2m_P}{mL}\boldsymbol{\phi}(vt) \tag{13}$$

$$\mathbf{C}_{11} = diag\left(2\xi_i\omega_i\right) \qquad i = 1,\ldots,N \tag{14}$$

$$\mathbf{C}_{21} = -c_P\boldsymbol{\phi}^T(vt) \tag{15}$$

$$\mathbf{K}_{11} = diag\left(\omega_i^2\right) \qquad i = 1,\ldots,N \tag{16}$$

$$\mathbf{K}_{21} = -k_P\boldsymbol{\phi}^T(vt) \tag{17}$$

$$\mathbf{Q}_B = P(t)\frac{2}{mL}\boldsymbol{\phi}(vt) \tag{18}$$

$$P(t) = m_P gr\sin\left(2\pi f_p t\right) \tag{19}$$

$$\boldsymbol{\phi}(vt) = \sin\frac{i\pi vt}{L} \qquad i = 1,\ldots,N \tag{20}$$

And c_P and k_P represent the pedestrian damping and stiffness parameters respectively.

3.4 *Comparison of SMD and MF models*

To determine the effect of the improved representation of pedestrians with the SMD model, a range of parameters were varied, and the 5-second root-mean-square (RMS) vibration response (R) noted. The results are examined through the non-dimensional ratio of the spring-mass-damper to moving force RMS results:

$$\mu = \frac{R_P^{SMD}}{R_P^{MF}} \tag{21}$$

To allow creation of a surface plot of the μ values, the pedestrian spring stiffness was varied from 10 to 35 kN/m and pedestrian mass was varied from 30 to 130 kg, again to fully explore possible values. The pedestrian damping ratio was kept constant at 0.3, and the pacing frequency and step length were 1.96 Hz and 0.66 m, respectively. The ratio of responses for each of these permutations was established for three bridge natural frequencies, 1.94 Hz, 2.0 Hz and 2.1 Hz. An illustration of the results is shown in Figure 4

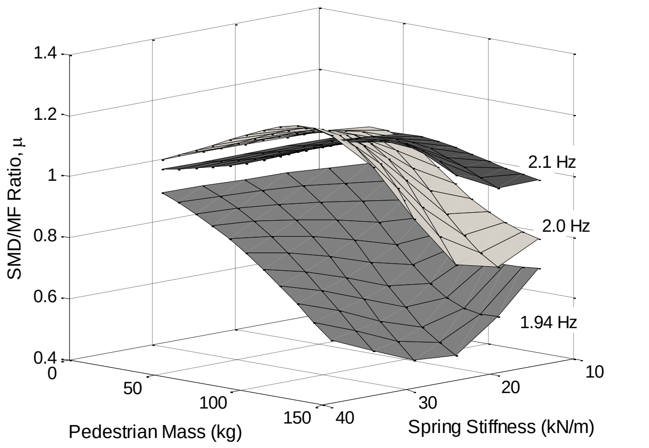
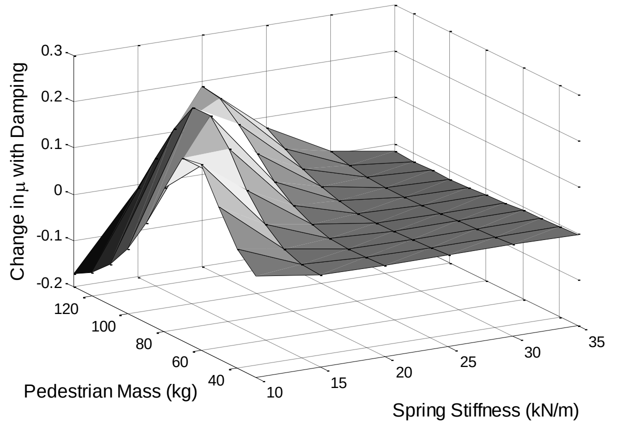


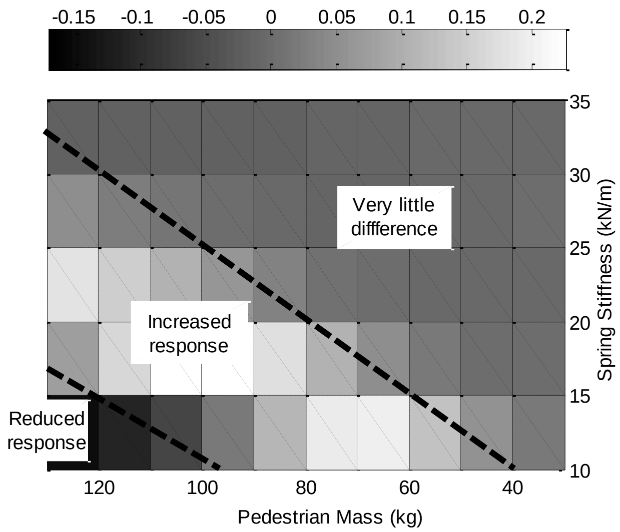
Figure 4. SMD-MF model RMS response ratio, μ.

From Figure 4, it can be seen that the moving force and spring-mass damper responses are similar for bridges with natural frequencies remote from the pedestrian pacing frequency of 1.96 Hz, when the pedestrian mass is below about 100 kg. However, it is quite clear that for low spring stiffnesses, and for heavier pedestrian mass, the SMD solution gives a lower response than the MF model. This is especially pronounced for a bridge with a natural frequency close to the pacing frequency.

In order to assess the influence that pedestrian damping has on the response, a pedestrian damping ratio of 0.1 was also examined for the critical bridge natural frequency of 1.94 Hz. The change in the response ratio, μ, as a result, is shown in Figure 5. Interestingly, reducing the damping increases the response for a particular combination of pedestrian mass and spring stiffness whilst for heavy mass, and low stiffness, the response is much reduced. However, in the main, for median to high stiffness and for typical pedestrian weights, the response is much the same ($\mu \approx 1$).



(a) Surface view;



(b) Contour view;

Figure 5. Change in response ratio, μ, with damping.

4 BRIDGE RESPONSE TO SINGLE PEDESTRIAN

4.1 Pedestrian Parameters

To determine the 95% characteristic 5-second RMS vertical mid-span acceleration response of the bridge to single pedestrian excitation, 1000 simulations of random individual pedestrians was performed. In these simulations, adult pedestrian mass was represented by a log-normal distribution with a mean of 73.9 kg and a coefficient of variation of 21.2% [25]. The stride length is taken to be normally distributed with a mean of 0.66 m [26], and a coefficient of variation of 10% is assumed. The pacing frequency is also considered to be normally distributed with a mean of 1.96 Hz and standard deviation of 0.209 Hz, based on derived meta-parameters from a literature survey as shown in Table 1.

Table 1. Pacing frequency: literature statistics

Ref	Mean (Hz)	SD* (Hz)	CoV**
[1]	2.0	0.173	0.087
[21]	2.0	0.13	0.065
[27]	1.9	0.25	0.13
[28]	1.83	--	--
[29]	1.8	--	--
[30]	2.2	0.3	0.14
Derived	1.96	0.209	0.1064

* Standard Deviation
** Coefficient of Variation

4.2 Characteristic response of single pedestrians

In performing these simulations, the moving force representation of a pedestrian was used, in keeping with design code practice, as outlined in Section 1.3. The resulting distribution and characteristic values are given in Figure 6. The characteristic response due to the moving force, R_P^{MF}, is defined here as the response below which 95% of samples are expected to fall, and is found in this case to have a value of

0.76 m/s^2 for the bridge with a natural frequency of 1.94 Hz. This is to be compared to the common rule used in BS 5400 [3], [4] of $0.5\sqrt{f_p}$ (which gives 0.7 m/s^2 in this case). As an aside, it was observed in another test of a modelled bridge with a natural frequency 2.38 Hz that the single pedestrian response reduces significantly to 0.27 m/s^2 due to the remoteness of the bridge natural frequency from the mean pacing frequency of 1.96 Hz.

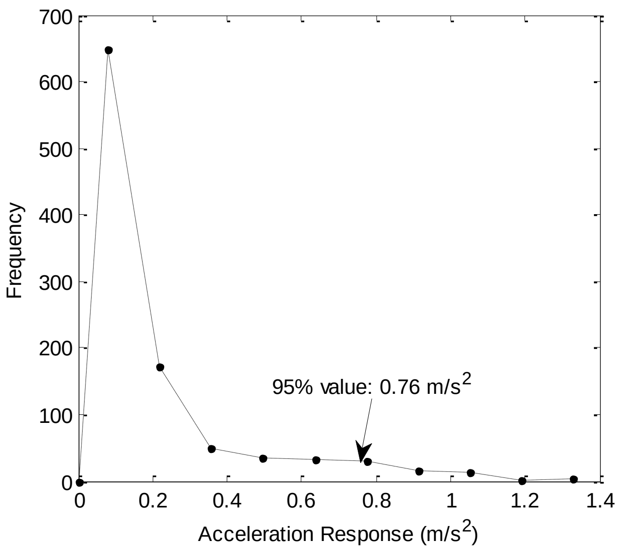


Figure 6. Distribution of 1.94 Hz bridge response for random single pedestrians (Moving Force model).

5 BRIDGE RESPONSE TO CROWD LOADING

5.1 Crowd properties and synchronization

A crowd length of 100 m and a width of 2 m was used to establish a representative crowd on the bridge at any point in time. The phase angle of the pedestrians is uniformly random in the interval 0 to 2π. Pedestrians' starting locations are based on a Poisson arrival process [1] and are thus described by the exponential distribution.

The level of synchronization within a crowd is reported with respect to the number of pedestrians on the bridge, N. Bachmann and Ammann [21] reported a synchronization level of $\sqrt{N}$ for a crowd density of 0.55 p/m^2 (where 'p' is the number of pedestrians) on a bridge with a natural frequency of 2.1 Hz. Grundmann et al [31] suggested a value of $0.135N$ for a bridge with a natural frequency of 1.94 Hz with a crowd density of 0.44 p/m^2. Whilst Fujino et al [32] found that 20% ($0.2N$) of the pedestrians were synchronized on a bridge with a frequency of 2.0 Hz with a very high crowd density of 2.11 p/m^2. To allow direct comparison with the published work, simulations were carried out to match the parameters used by those authors. As a result, crowd densities of 0.44 p/m^2, 0.55 p/m^2, and 2.11 p/m^2, were simulated on bridges with natural frequencies of 1.94 Hz, 2.1 Hz and 2.0 Hz respectively. To model the SMD in the crowd situation, the pedestrian stiffness (k_P) is taken to be normally distributed with a mean of 22.5 kN/m with a standard deviation of 2.25 kN/m. This was chosen as it is the midpoint of the values quoted by Lee and Farley [9]. All pedestrians are considered to have a non-stochastic damping ratio of 0.3, as found by Zhang et al [11]. The pedestrian weight was, as in Section 4.1, represented by a log-normal distribution with a mean of 73.9 kg and a coefficient of variation of 21.2% [25].

The proportion of pedestrians taken to be synchronized was also chosen to coincide with the reported synchronization levels in [21], [31] and [32]. In addition to this, six other synchronization proportions are investigated on the bridge studied by Grundmann et al [31], which is closest to the mean pacing frequency as outlined in Section 4.1. The level of synchronization presented here ranges from 0 to 100% and incorporates the values presented by [21], [31] and [32]. The pedestrians deemed to be synchronized are given the same pacing frequency and phase angle. These parameters are randomly selected according to their respective distributions previously given. The synchronized pedestrians are randomly distributed throughout the crowd. It is acknowledged that this is a simplification as some clusters of synchronized pedestrians may occur, but this is not considered here. For the case of no enforced synchronization, it is still statistically possible that very low levels of synchronization may yield similar results.

Enhancement factors for levels of synchronization higher than that quoted by Fujino et al [32] have not been found in the literature. High levels of synchronization in a crowd may be typical of a marching band or army troops, where levels close to 100% may be expected.

5.2 Crowd modelling

Due to the complexities of using the modal approach (Section 3.3), to analyse the bridge response to a crowd of pedestrians modelled as SMDs, a moving force model was retained for modelling the crowd. However, the force used to represent an individual pedestrian is altered to account for the relationship between the SMD results and the MF results discussed in Section 3.4. Thus, the force applied by a pedestrian in the crowd model is:

$$P(t) = m_P g\left[\mu\left(m_P, k_P\right)\right]\left[1 + r\sin\left(2\pi f_p t\right)\right] \qquad (21)$$

In this expression, the functional relationship between the SMD and MF model for different pedestrian mass and stiffnesses, $\mu\left(m_P, k_P\right)$, given as the surfaces of Figure 4 for the particular bridge frequencies is used. This results in a moving force model that replicates the results of an SMD model for each pedestrian comprising the crowd. Linear interpolation is used for masses and stiffnesses between calculated points on the $\mu\left(m_P, k_P\right)$ surface in determining μ for each pedestrian.

5.3 Characteristic crowd response

A typical crowd response using the above modelling strategies is given in Figure 7. This response is for a crowd density of 0.44 p/m^2 with a synchronization of $0.135N$ on a bridge with a natural frequency of 1.94 Hz. Figure 7(a) shows the midspan acceleration and the 5-second RMS against time. Figure 7(b) shows the number of pedestrians on the bridge, the time at which they enter and exit the bridge and highlights the pedestrians that are synchronized. For each crowd simulated the peak RMS vertical response is noted.

For the crowd densities of 0.44 p/m^2, 0.55 p/m^2 and 2.11 p/m^2, with their specified level of synchronization, 1000 sample crowd responses were determined. The characteristic response (the 95-percentile) was then determined for the crowd loading scenario associated with each bridge frequency, crowd density, and level of synchronization considered. The corresponding enhancement factors are determined from Equation (23) with the values of characteristic single pedestrian response, R_P^{MF}, found previously as 0.76 m/s^2 for 1.94 Hz bridge (Figure 6). Values of 0.85 m/s^2 and 0.84 m/s^2 were obtained for the bridges with a natural frequency of 2.0 Hz and 2.1 Hz respectively.

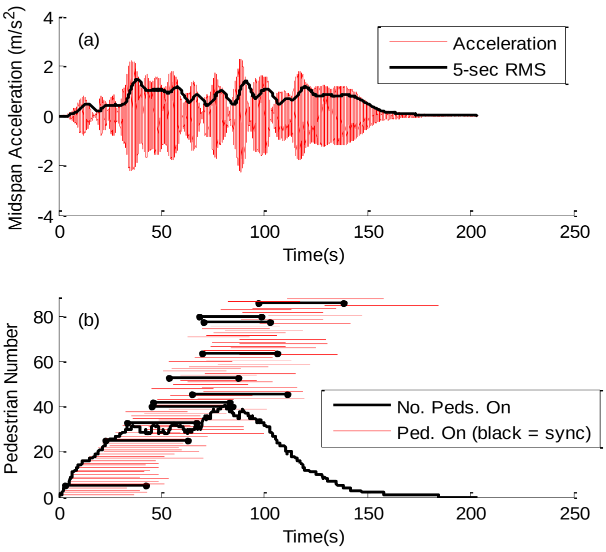


Figure 7. (a) Typical crowd response; (b) crowd diagnostics. Beam: 1.94 Hz, Density: 0.44 p/m^2, Sync: 0.135N.

6 CROWD LOADING ENHANCEMENT FACTORS

6.1 *Enhancement factors based on characteristic responses*

Following investigations into the enhancement factors used by Matsumoto et al [1], the crowd loading enhancement factor is defined as:

$$m = \frac{R_C^{SMD}}{R_P^{MF}} \tag{21}$$

In which R_C^{SMD} is the response due to the crowd based on an SMD model, and R_P^{MF} is the single pedestrian response, based on a moving force model. With m known to designers, the bridge response due to a crowd can be estimated from that of a single pedestrian, with m estimated using a high-fidelity SMD model for the crowd response.

6.2 *Comparison to previous literature*

In Figure 8, enhancement factors for a crowd density of 0.55 p/m^2 on the 1.94 Hz bridge, with varying levels of synchronization, are presented. These are also compared to previous work carried out by the present authors [33] which used a moving force model to represent pedestrians in the crowd. It can be seen that there is a significant reduction in the enhancement factors derived, depending on the models used for the individual pedestrians that comprise the crowd. Thus it is possible that significant over-estimation of crowd-induced vibrations can result from solely using moving force models.

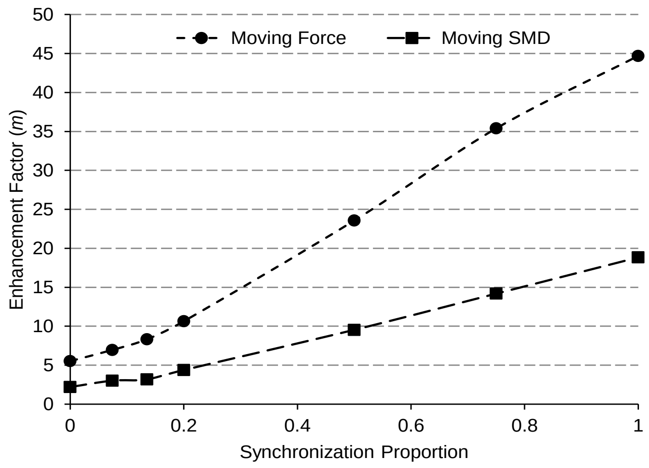


Figure 8. MF vs. SMD Crowd at Varying levels of Synchronization

Figure 9 compares the enhancement factors obtained in this work to those of:

1. Bachmann and Ammann [21], who examined a density of 0.55 p/m^2, with synchronization of ($\sqrt{N}$)% (where N is the number of pedestrians on the bridge), for a bridge of 2.1 Hz;
2. Grundmann et al [31], who used a density of 0.44 p/m^2, with synchronization of 13.5% for a bridge of 1.94 Hz;
3. Fujino et al [32], who found a very high density of 2.11 p/m^2, with a synchronization of 20%, for a bridge of second natural frequency, 2.0 Hz.

The present results show good correspondence with the work of these authors at the specified levels of synchronization, for the same bridge frequencies. The enhancement factors found from this work are all lower than those found by these authors. This is to be expected since the present work has found that SMD pedestrian models result in lower bridge response when the bridge natural frequency is near the pedestrian pacing frequency.

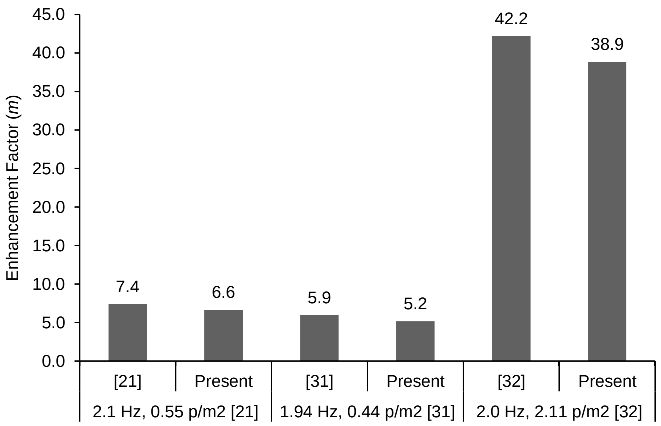


Figure 9. Comparison of enhancement factors.