# Products End-of-Life Scenario Modelling Using Linear Algebra

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## Abstract

There are several options at the end-of-life (EOL) of a product: reuse/part reclamation, remanufacturing, recycling, incineration (with or without energy recovery) or disposal to landfill. Each option has an economic, an environmental and a social impact. The impacts are depicted by indicators that have specific values for each option. A set of indicators' values specific to an EOL option represents the components of that EOL option's vector. As the three dimensions (environmental economic and social) of each option are important, one vector for each dimension needs to be considered. A generic EOL scenario, which may comprise a combination of EOL options can be expressed as a linear combination of options. Therefore, considering the percentages of a product reaching each EOL option as scalars, and the vectors associated to the options, vectors associated to the whole scenario can be calculated. This modelling approach permits the use of linear algebra tools in expressing and solving problems related to EOL scenario of products. Such a model can be applied to waste from electrical and electronic products. It represents the foundation in designing the structure of transaction-processing information systems. The model provides not only information about the environmental, economic and social impact of a certain EOL scenario, by applying the linear programming theory, it can also calculate the best scenario structure given the targets set by different laws and regulations such as the European Union Waste Electrical and Electronic Equipment (EU WEEE) Directive and the EOL options vectors. The model can be also used as a 'What If' support tool for decision-making showing possible results as a consequence of the influence of different factors (e.g. change of technology, different quantity of products, etc.).

## Introduction

Resource recovery, the recovery of functional or material value from products at end-of-life, is an area currently receiving considerable attention from policy makers, from product manufacturers, from engineering researchers and from general public [1]. At all levels, national and international, concepts such as sustainable development and polluter pays principle are being taken into account in setting strategies and plans for further change and development.

The "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" also referred to as sustainable development and "polluters have to bear the costs of meeting governmental requirements set in relation to desired environmental standards" also known as the polluter pays principle are basic principles envisaging all the phases comprised in a product lifecycle [1]. As end-of-life is the last phase of a product lifecycle, it is necessary to address end-of-life issues bearing in mind the mentioned principles – sustainable development and polluter pays principle.

Environmental laws that put pressure on firms to take back their products and take care of further treatment are becoming a reality. So is the Draft Directive of the European Parliament and of the Council on waste electrical and electronic equipment (WEEE) which sets targets, players and responsibilities for the "collection, treatment and environmentally sound disposal" of WEEE [2]. In order to ameliorate the environmental performances of electrical and electronic products the WEEE Directive puts pressure on the last phase of the lifecycle of the product but consequently all the other phases in the lifecycle are also impacted since design, manufacturing processes, useage influence the choice of EOL treatment.

This document is structured in three sections: the first section presents the possible options for a product reaching its end-of-life stage; the second section shows the environmental, economic and social indicators that describe best the environmental, economic and social impact of the end-of-life treatment option (these indicators form the vectors used in the model); the last section describes the mathematical model for calculating the impacts of different EOL options and the use of the model as a 'what if' support tool for decision-making.

## End of Life Options

The treatment options that an electrical or electronic product may undergo at its end-of-life stage are: re-use/ part reclamation, remanufacturing, recycling, incineration with or without energy recovery and disposal to landfill. As incineration without energy recovery is not considered a viable option for EOL in Ireland, only incineration with energy recovery will be considered in this paper.

According to the Draft WEEE Directive, *reuse* means any operation by which a whole product or its components, having reached their end-of-life, are used for the same purpose for which they were conceived [2]. The reuse of a product may be the reuse of the entire product, for example the selling of second hand cars or computers, or it may be the reuse of components of a product, for spares for example. The most effective solution to the sustainement of the resources that comprise a product is to reuse it at end of life [1]. Through extension of the product life cycle in this way all the material, human, energy and process resources are sustained through continued use of the full product functionality.

*Remanufacturing* is an environmentally and economically sound way to achieve many of the goals of sustainable development. Remanufacturing focuses on value-added recovery, rather than just materials recovery (recycling) [3]. The United States Environmental Protection Agency considers remanufacturing as an "integral foundation of reuse activities and reports that less energy is used and less waste is produced with this type of activities" [4]. Lund defines *remanufacturing* as "an industrial process in which worn-out products are restored to like-new condition. Through a series of industrial processes in a factory environment, a discarded product is completely disassembled. Useable parts are cleaned, re-furbished, and put into inventory. Then the new product is reassembled from the old and, where necessary, new parts to produce a fully equivalent - and sometimes superior - in performance and expected lifetime to the original new product" [5].

*Recycling* is defined by the WEEE Directive as the reprocessing in a production process of the waste materials for the original purpose or for other purposes, but excluding energy recovery [2]. Recycling is performed to retrieve the material content of the used and non-functioning products. Recycling represents a process by which products otherwise destined for disposal are processed to recover base materials [1]. The recovered material represents little of the human, energy and process resource inputs, but allows the material properties to be sustained [1].

The ultimate end-of-life options are *incineration with energy recovery* or simply *discard to landfill*. *Incineration with energy recovery* is the controlled burning of wastes at high temperatures in a facility designed for efficient and complete combustion [6]. Landfilling is a long-used practice of depositing waste in a dump site at the outskirts of a community. Disposal without any material or energy recovery is to be regarded as a matter of last resort.

Each of the end-of-life options presented has an impact on environment, an impact on community and economic implications as well. A literature review has been carried out in order to identify these implications and the possibility of quantifying them. Environmental, economic and social indicators have been studied and some of those chosen for the model described in this document.

## Economic, Environmental and Social Vectors Associated to EOL Options

Sustainability encompasses three basic areas: environmental protection, economic development and social equity. One of the key goals of sustainability is that companies should provide more value for their products and services while seeking to transform less materials and energy and to reduce environmental influences.

In order to measure progress towards sustainable goals of a company, a set of indicators must be calculated. They must include information on the firm's economic performance, as well as its environmental and social performance [7]. Figure 1 presents the measurement of the three dimensions of sustainability.

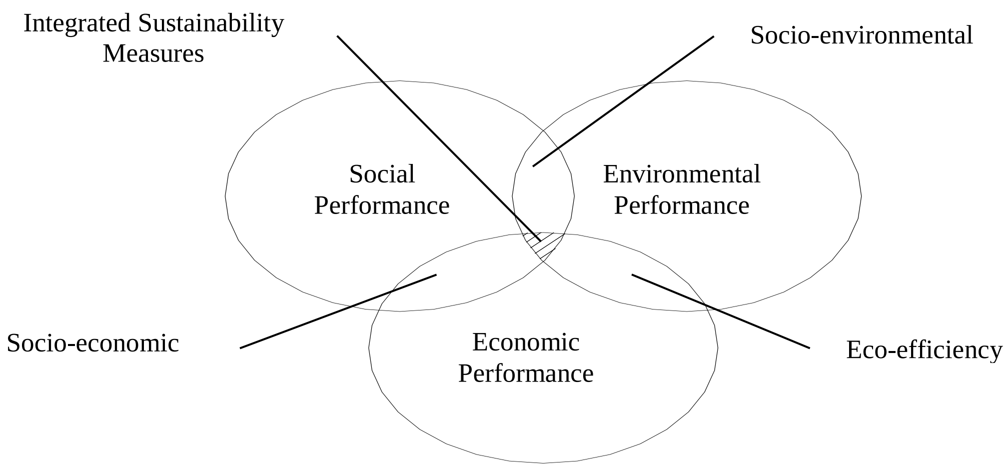


Figure 1. Sustainability measurements [7]

**The Environmental Vector**

The impact of any company activity on environment is vast but it can be classified into three main categories: damage to resources, damage to ecosystem and damage to human health [8], [9]. There is a great variety of environmental indicators in use that show these impacts. The main categories of indicators that show environmental performance are: (1) materials use – indicator that tracks resource inputs; (2) energy consumption – quantity of energy used; (3) non-product output – quantities of waste generated before recycling, treatment or disposal; (4) pollutant releases – quantities of pollutants released to air, water and land [7].

For our model we have chosen the following environmental indicators:

- *Environmental indicators that show damage to resources*:
  - Non-renewable primary energy input
  - Materials consumption
  - Water consumption
- *Environmental indicators that show damage to ecosystem*:
  - Greenhouse effect
  - Ozone layer depletion
  - Acidification
  - Water nutrient pollution
- *Environmental indicators that show damage to human health*:
  - Hazardous substances emitted into air
  - Hazardous substances emitted into water
  - Emission of carcinogenic substances

As the WEEE Directive will force producers to consider different targets for the rate of recovery of their products, another set of indicators was considered for our model:

- *Environmental indicators that show compliance with regulation*:
  - Percentage of waste that is reused/remanufactured/recycled/incinerated/landfilled
  - Average percentage of product that is reused/remanufactured/recycled/incinerated/ landfilled

Given a product and considering that the whole product (100%) is subject to one EOL treatment option, for each EOL option the values of these indicators will be calculated. In our model, for a certain EOL option these values represent the coordinates of the environmental vector for that particular EOL option.

Therefore, there will be five environmental vectors, one for each EOL option. If we use column matrix notation for the vectors, for reuse/part reclamation we'll have the following vector:

$$\mathbf{v}_{reuse} = \begin{bmatrix} v_{reuse_1} \\ v_{reuse_2} \\ \vdots \\ v_{reuse_n} \end{bmatrix}$$

where $v_{reuse_1}, \ldots, v_{reuse_n}$ are the values of the environmental indicators considered in the model
$n$ = number of indicators considered in the model (12 in our case)

Similarly we can get $\mathbf{v}_{remanufacturing}$, $\mathbf{v}_{recycling}$, $\mathbf{v}_{incineration}$, $\mathbf{v}_{landfill}$.

**The Economic Vector**

Economic indicators give a very good image of the business and all decision-makers are familiar with their meaning. Indicators like net sales, profit, costs, cash flow, gross profit margin, return on investment or debt ratio are relevant for any business.

For the companies involved in activities at the end-of-life of products cost is an important issue. Although producers will be obliged by law to recover their products, processing cost will be a prime issue when choosing the end-of-life option for their products. That is the reason why we chose the costing elements as economic indicators for our model. As it is very important as well to know the constitution of the cost, the following components of the processing cost are included in the model:

- Direct material costs
- Direct labour costs
- Production overheads

Given a product and considering that the whole product (100%) is subject to one EOL treatment option, for each EOL option the values of these indicators will be calculated. In our model, for a certain EOL option these values represent the coordinates of the economic vector for that particular EOL option.

Therefore, there will be five economic vectors, one for each EOL option. If we use column matrix notation for the vectors, for reuse/part reclamation we'll have the following vector:

$$\mathbf{w}_{reuse} = \begin{bmatrix} w_{reuse_1} \\ w_{reuse_2} \\ \vdots \\ w_{reuse_m} \end{bmatrix}$$

where $w_{reuse_1}, \ldots, w_{reuse_m}$ are the values of the economic indicators considered in the model
$m$ = number of indicators considered in the model (3 in our case)

Similarly we can get $\mathbf{w}_{remanufacturing}$, $\mathbf{w}_{recycling}$, $\mathbf{w}_{incineration}$, $\mathbf{w}_{landfill}$.

**The Social Vector**

As stated before, sustainable development is built on three pillars: economic growth, ecological balance and social progress, all of which are integral to the achievement of sustainable development [10]. Therefore corporate social responsibility is business's contribution to the third pillar of sustainable development: social progress [10]. The social performance of a business measures its contribution to the social progress. Social performance of a company actually measures the business performance in relation to its impact on different stakeholder groups (communities, employees, suppliers, etc.) [7].

Corporate social performance indicators can be developed based on the general social issues and the stakeholders of companies. Some key social elements are: employment practices (the provision of a safe working environment, freedom from discrimination on race, gender, colour or creed, etc.), community relations (the contribution of a firm to community development e.g. job creation, etc.), ethical sourcing (ensure that suppliers do not use child or forced labour, provide safe working conditions and fair wages, etc.) and the usefulness of products to society (the contribution of products and services to social welfare, equity, etc.) [7].

We have chosen the following social indicators for our model:

- Number of jobs created
- Number of complaints from community
- Degree of resources applied to support community environmental programs
- Degree of compliance with regulations

Given a product and considering that the whole product (100%) is subject to one EOL treatment option, for each EOL option the values of these indicators will be calculated. In our model, for a certain EOL option these values represent the coordinates of the social vector for that particular EOL option.

Therefore, there will be five social vectors, one for each EOL option. If we use column matrix notation for the vectors, for reuse/part reclamation we'll have the following vector:

$$\mathbf{u}_{reuse} = \begin{bmatrix} u_{reuse_1} \\ u_{reuse_2} \\ \vdots \\ u_{reuse_k} \end{bmatrix}$$

where $u_{reuse_1}, \dots, u_{reuse_k}$ are the values of the social indicators considered in the model

$k$ = number of indicators considered in the model (4 in our case)

Similarly we can get $\mathbf{u}_{remanufacturing}$, $\mathbf{u}_{recycling}$, $\mathbf{u}_{incineration}$, $\mathbf{u}_{landfill}$.

## EOL Scenario Modelling

So far we have considered that a waste product reaching its end-of-life stage will be 100% reused or remanufactured or recycled or incinerated or landfilled. But the EOL scenario may be a combination of all these or just of some of them. Maybe only a component of the product can be reused and another recycled and the rest of it landfilled. Usually, it is not the same processor carrying out reuse, remanufacturing, recycling, incineration and landfill at the same time. Therefore, data regarding the EOL treatment of a certain product comes from different sources.

Legislation, including the WEEE Directive, will necessitate the producers interest in all the three dimensions of the impact of his/her EOL product, whatever the combination of EOL treatment it may be subject to. The model presented here calculates the environmental, economic and social impacts of a product subject to a generic EOL scenario treatment that can be any combination of the five options (reuse/part reclamation, remanufacturing, recycling, incineration with energy recovery and landfill).

Let us consider an EOL scenario for a product as follows:

- $p_1$ - % of the product's weight that is reused
- $p_2$ - % of the product's weight that is remanufactured
- $p_3$ - % of the product's weight that is recycled
- $p_4$ - % of the product's weight that is incinerated with energy recovery
- $p_5$ - % of the product's weight that is landfilled

The environmental impact of this product is given by a vector, denoted as $\mathbf{v}_{EOL}$, which is a linear combination of the environmental vectors of the five EOL options ($\mathbf{v}_{reuse}$, $\mathbf{v}_{remanufacturing}$, $\mathbf{v}_{recycling}$, $\mathbf{v}_{incineration}$, $\mathbf{v}_{landfil}$) with weights $p_1$, $p_2$, $p_3$, $p_4$, $p_5$.

$$\mathbf{v}_{EOL} = p_1\mathbf{v}_{reuse} + p_2\mathbf{v}_{remanufacturing} + p_3\mathbf{v}_{recycling} + p_4\mathbf{v}_{incineration} + p_5\mathbf{v}_{landfill}$$

Or, if we use matrices, the same result will be obtained with the equation:

$$\mathbf{V}\text{x}\mathbf{p} = \mathbf{v}_{EOL}$$

where $\mathbf{V} = [\mathbf{v}_{reuse}\ \ \mathbf{v}_{remanufacturing}\ \ \mathbf{v}_{recycling}\ \ \mathbf{v}_{incineration}\ \ \mathbf{v}_{landfil}]$ = environmental matrix of product

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_5 \end{bmatrix}$$ = structural vector of EOL scenario

Or

$$\begin{bmatrix} v_{reuse_1} & v_{remanufacturing_1} & v_{recycling_1} & v_{incineration_1} & v_{landfill_1} \\ v_{reuse_2} & v_{remanufacturing_2} & v_{recycling_2} & v_{incineration_2} & v_{landfill_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{reuse_n} & v_{remanufacturing_n} & v_{recycling_n} & v_{incineration_n} & v_{landfill_n} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_5 \end{bmatrix} = \begin{bmatrix} v_{EOL_1} \\ v_{EOL_2} \\ \vdots \\ v_{EOL_n} \end{bmatrix}$$

Similarly, the economic and social vectors associated to the EOL scenario could be laid down. The economic vector:

$$\mathbf{w}_{EOL} = p_1\mathbf{w}_{reuse} + p_2\mathbf{w}_{remanufacturing} + p_3\mathbf{w}_{recycling} + p_4\mathbf{w}_{incineration} + p_5\mathbf{w}_{landfill}$$

Or, using matrices:

$$\mathbf{W}\text{x}\mathbf{p} = \mathbf{w}_{EOL}$$

The social vector:

$$\mathbf{u}_{EOL} = p_1\mathbf{u}_{reuse} + p_2\mathbf{u}_{remanufacturing} + p_3\mathbf{u}_{recycling} + p_4\mathbf{u}_{incineration} + p_5\mathbf{u}_{landfill}$$

Or, using matrices:

$$\mathbf{U}\text{x}\mathbf{p} = \mathbf{u}_{EOL}$$

The model can be also used for solving a 'what if' situation. For instance, if the decision-maker (producer, processor) considers product design changes (producer), technology changes (processor) or any other change in activity, he/she might be interested in finding the total environmental or economic or social impact of the product in a given EOL scenario. Such changes might alter entries in the environmental or the economic or the social matrix of our model and, consequently, the total impact of the product subject to the given EOL scenario.

Another possible application of the model is the calculation of the best EOL scenario from an environmental or economic or social point of view, given some environmental, economic or social constraints.

Different laws and regulations set maximum or minimum values for different environmental indicators (maximum emissions of $CO_2$, minimum quantity of products recycled). At the same time people funding different treatment of EOL products could afford up to a maximum amount of money (economic constraint). These facts are also referred to as constraints and they are given as maximum or minimum values for different indicators denoted $c_{env1}$, $c_{env2}$,…, $c_{envn}$ (for the environmental indicators), $c_{ec1}$, $c_{ec2}$,..$c_{ecm}$ (for the economic indicators) and $c_{soc1}$, $c_{soc2}$,… $c_{socp}$ (for social indicators).

Given an EOL scenario of a product depicted by the environmental, economic and social indicators and considering the constraints on those indicators the following (in)equations may be written:

$$v_{EOL1} (\leq)(=)(\geq)\, c_{env1},\ v_{EOL2} (\leq)(=)(\geq)\, c_{env2},\ldots,\ v_{EOLn} (\leq)(=)(\geq)\, c_{envn}$$
$$w_{EOL1} (\leq)(=)(\geq)\, c_{ec1},\ v_{EOL2} (\leq)(=)(\geq)\, c_{ec2},\ldots,\ v_{EOLm} (\leq)(=)(\geq)\, c_{ecm}$$
$$u_{EOL1} (\leq)(=)(\geq)\, c_{soc1},\ u_{EOL2} (\leq)(=)(\geq)\, c_{soc2},\ldots,\ v_{EOLn} (\leq)(=)(\geq)\, c_{socp}$$

Considering the formulae given by our model we'll get the following system of (in)equations (1):

$$\begin{cases}
v_{reuse_1} \cdot p_1 + v_{remanufacturing_1} \cdot p_2 + v_{recycling_1} \cdot p_3 + v_{incineration_1} \cdot p_4 + v_{landfill_1} \cdot p_5 (\leq)(=)(\geq) c_{env_1} \\
v_{reuse_2} \cdot p_1 + v_{remanufacturing_2} \cdot p_2 + v_{recycling_2} \cdot p_3 + v_{incineration_2} \cdot p_4 + v_{landfill_2} \cdot p_5 (\leq)(=)(\geq) c_{env_2} \\
\vdots \\
v_{reuse_n} \cdot p_1 + v_{remanufacturing_n} \cdot p_2 + v_{recycling_n} \cdot p_3 + v_{incineration_n} \cdot p_4 + v_{landfill_n} \cdot p_5 (\leq)(=)(\geq) c_{env_n} \\
\\
w_{reuse_1} \cdot p_1 + w_{remanufacturing_1} \cdot p_2 + w_{recycling_1} \cdot p_3 + w_{incineration_1} \cdot p_4 + w_{landfill_1} \cdot p_5 (\leq)(=)(\geq) c_{ec1} \\
w_{reuse_2} \cdot p_1 + w_{remanufacturing_2} \cdot p_2 + w_{recycling_2} \cdot p_3 + w_{incineration_2} \cdot p_4 + w_{landfill_2} \cdot p_5 (\leq)(=)(\geq) c_{ec_2} \\
\vdots \\
v_{reuse_m} \cdot p_1 + v_{remanufacturing_m} \cdot p_2 + v_{recycling_m} \cdot p_3 + v_{incineration_m} \cdot p_4 + v_{landfill_m} \cdot p_5 (\leq)(=)(\geq) c_{ec_m} \\
\\
u_{reuse_1} \cdot p_1 + u_{remanufacturing_1} \cdot p_2 + u_{recycling_1} \cdot p_3 + u_{incineration_1} \cdot p_4 + u_{landfill_1} \cdot p_5 (\leq)(=)(\geq) c_{soc_1} \\
u_{reuse_2} \cdot p_1 + u_{remanufacturing_2} \cdot p_2 + u_{recycling_2} \cdot p_3 + u_{incineration_2} \cdot p_4 + u_{landfill_2} \cdot p_5 (\leq)(=)(\geq) c_{soc_2} \\
\vdots \\
u_{reuse_k} \cdot p_1 + u_{remanufacturing_k} \cdot p_2 + u_{recycling_k} \cdot p_3 + u_{incineration_k} \cdot p_4 + u_{landfill_k} \cdot p_5 (\leq)(=)(\geq) c_{soc_n}
\end{cases} \quad (1)$$

The WEEE Directive aims maximisation of recovery which is the maximisation of reuse, remanufacturing and recycling of products, or:

$$z = p_1 + p_2 + p_3 = \max$$

In other words the problem can be reformulated as follows:

Find the best scenario for an EOL of a product ($p_1$, $p_2$, $p_3$, $p_4$, $p_5$) taking into account constraints set by different laws and regulations aiming to get maximum of the product recovered (z=max) and $p_1$, $p_2$, $p_3$, $p_4$, $p_5$ are all greater then zero.

This type of problem is a linear programming problem. The linear function in z is the objective function and the (in)equations from the system are the constraints. These constraints plus the non-negativity constraints give us the feasible region. The feasible solution which gives the maximum value to the objective function is the solution we are looking for [11] [12] [13] [14].

## Conclusions

Considering an EOL scenario as a linear combination of EOL options gives us the possibility of using the linear algebra tools in order to solve decision problems related to EOL of products. To make this model work, transaction-processing information systems should be put in place in order to capture data needed to calculate the entries for the environmental, economic and social matrices for the (WEEE Directive recommends establishing databases on WEEE and its treatment). Also methodologies to calculate such values should be developed.

The accuracy in using our model depends very much on the quality of data gathered into transaction-processing information systems. Tracking and tracing the EOL scenarios for products is vitally important in order to get accurate values for the entries of the product matrices and for the structural vectors of EOL scenarios.

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